2016 東邦大(医)

1

$f(x)=4\log(6+\sqrt{9+x^3})$

$f'(x)$

$=4\cdot\dfrac{\frac{1}{2}(9+x^3)^{-\frac{1}{2}}3x^2}{6+\sqrt{9+x^3}}$

$=\dfrac{6x^2}{6\sqrt{9+x^3}+9+x^3}$

$f'(3)$

$=\dfrac{54}{36+36}=\underline{\dfrac{3}{4}}$ //

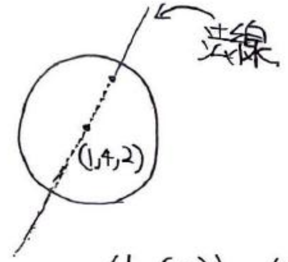
2

$(x-1)^2-1+(y-4)^2-16$

$+(z-2)^2-4-28=0$

$\Leftrightarrow (x-1)^2+(y-4)^2+(z-2)^2=49$

中心 $\underline{(1,4,2)}$ // 半径 $\underline{7}$ //



法線ベクトル $\begin{pmatrix}1-(-5)\\4-6\\2-5\end{pmatrix}=\begin{pmatrix}6\\-2\\-3\end{pmatrix}$ で

$(-5,6,5)$ を通る平面は

$6(x+5)-2(y-6)-3(z-5)=0$

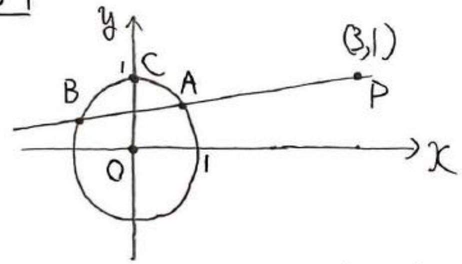
$\Leftrightarrow 6x-2y-3z+57=0$

$\downarrow$ z軸との交点は $x=y=0$ を代入

$-3z+57=0$ $\therefore z=19$

$\therefore$ 交点 $(0,0,\underline{19})$ //

3



$C(0,1)$ とおくと方べきの定理より

$PA\cdot PB=PC^2$

$\Leftrightarrow \sqrt{5}\cdot PB=9$

$\therefore PB=\dfrac{9\sqrt{5}}{5}$

$AB=PB-PA$

$=\dfrac{9}{5}\sqrt{5}-\sqrt{5}$

$=\underline{\dfrac{4\sqrt{5}}{5}}$ //

余弦定理より

$\cos\angle AOB=\dfrac{1^2+1^2-AB^2}{2\cdot1\cdot1}$

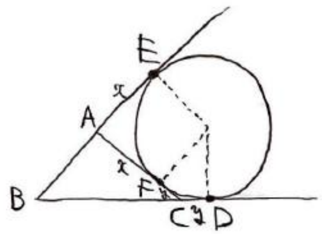
$=1-\dfrac{8}{5}$

$=-\dfrac{3}{5}$

$\triangle OAB=\dfrac{1}{2}\cdot1\cdot1\cdot\underbrace{\sin\angle AOB}_{\frac{4}{5}}$

$=\underline{\dfrac{2}{5}}$ //

4



図のように頂点をとり

$AE=AF=x,\ CF=CD=y$

とする.

$\begin{cases}x+y=24\\19+x=27+y\end{cases}$

$\Leftrightarrow \begin{cases}x+y=24\\x-y=8\end{cases}$

$\therefore x=16,\ y=8$

$\therefore BD=27+8=\underline{35}$ //

5

$1-\sin\theta=u$ とおくと

$4(1+\sin\theta)-\dfrac{3}{1-\sin\theta}$

$=4(2-u)-\dfrac{3}{u}$

$=8-\left(4u+\dfrac{3}{u}\right)$ $\quad 4u>0,\ \dfrac{3}{u}>0$

(相加平均) $\geqq$ (相乗平均)

$\leqq 8-2\sqrt{4u\cdot\dfrac{3}{u}}$

$=8-4\sqrt{3}$

$=\underline{-4\sqrt{3}+8}$ //

等号成立は $4u=\dfrac{3}{u}$

$\therefore u=\dfrac{\sqrt{3}}{2}$

$\therefore \sin\theta=1-\dfrac{\sqrt{3}}{2}$ のとき

6

$y = a\left(x^2 + \frac{b}{a}x\right) + c$

$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2-4ac}{4a}$

$m = -\frac{b^2-4ac}{4a} > \frac{11}{2}$

$\Leftrightarrow b^2 - 4ac < -22a$

$\Leftrightarrow b^2 < 2(2c-11)a$

$\downarrow c=6$

$b^2 < 2a$

a	b	a	b
6	3	4	2
	2		1
	1	3	2
5	3		1
	2	2	1
	1	1	1

$P\left(m > \frac{11}{2}\right) = \frac{12}{6^2} \cdot \frac{1}{6} = \frac{1}{18}$ //

7

$x + x^{104} = (1 - x + x^2)Q(x) + ax + b$

とおく.

$x^2 - x + 1 = 0 \Leftrightarrow x = \frac{1 \pm \sqrt{3}i}{2}$

$x = \frac{1+\sqrt{3}i}{2} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

を代入すると

$\frac{1+\sqrt{3}i}{2} + \cos\frac{104}{3}\pi + i\sin\frac{104}{3}\pi$

$= \frac{1+\sqrt{3}i}{2} + \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi$

$= \sqrt{3}i = 0 + \frac{a}{2} + b + \frac{\sqrt{3}}{2}ai$

$\therefore a = 2 \quad b = -1$

$\therefore$ 余り $-1 + 2x$ //

8

$f(x) = \frac{2}{3}\log x + 2x^2 + ax \quad (x>0)$

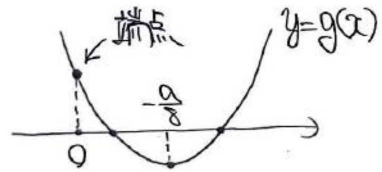
$f'(x) = \frac{2}{3x} + 4x + a$

$= \frac{12x^2 + 3ax + 2}{3x}$

$f(x)$が極値をもつには

$g(x) = 12x^2 + 3ax + 2 = 0$が $x>0$

で異なる2つの実数解をもてばいい.



$\begin{cases} \text{端} & g(0) = 2 > 0 \\ \text{軸} & -\frac{a}{8} > 0 \\ \text{判} & D = 9a^2 - 96 > 0 \end{cases}$

$\therefore \begin{cases} a < 0 \\ a^2 > \frac{32}{3} \end{cases}$

$\therefore a < -\frac{4\sqrt{2}}{\sqrt{3}} = -\frac{4\sqrt{6}}{3} = \frac{-4\sqrt{6}}{3}$ //

9

$\angle DCA = 15°$ より

$\angle BDC = 30° + 15° = 45°$

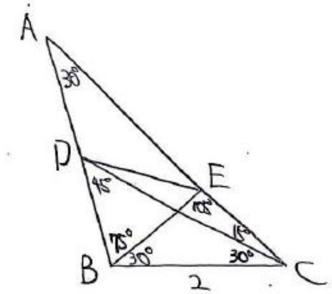
正弦定理より

$\frac{BC}{\sin 45°} = \frac{CD}{\sin 105°}$

$\therefore CD = 2 \cdot \sqrt{2} \cdot \sin(60° + 45°)$

$= 2\sqrt{2} \cdot \frac{\sqrt{6}+\sqrt{2}}{4}$

$= \sqrt{3} + 1$ //



$\triangle BCE$に正弦定理

$\frac{2}{\sin 105°} = \frac{CE}{\sin 30°}$

$\therefore CE = 2\sin 30° \cdot \frac{1}{\sin 105°}$

$= \frac{4}{\sqrt{6}+\sqrt{2}}$

$= \sqrt{6} - \sqrt{2}$

$\triangle DCE$に余弦定理

DE^2

$= CD^2 + CE^2 - 2(\sqrt{3}+1)\sqrt{2}(\sqrt{3}-1)\cos 15°$

$= 4 + 2\sqrt{3} + 2(4 - 2\sqrt{3}) - 4\sqrt{2}\,\frac{\sqrt{6}+\sqrt{2}}{4}$

$= 12 - 2\sqrt{3} - (2\sqrt{3} + 2)$

$= 10 - 4\sqrt{3} = -4\sqrt{3} + 10$ //

10

$(a+1)x^2 + (10y - 2a)x - 3y^2 - 12y + a = 0$

としたときに

$x = \frac{-5y + a \pm \sqrt{\frac{D}{4}}}{a+1} \quad (a \neq -1)$

となるので$\frac{D}{4}$が平方式でなければならない. つまり

$\frac{D}{4} = (5y - a)^2 - (a+1)(-3y^2 - 12y + a)$

$= (28 + 3a)y^2 + (2a + 12)y - a = 0$

が重解をもてば(1)ので

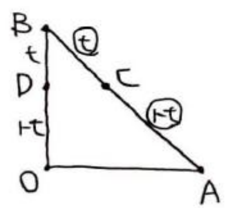
$(a+6)^2-(28+3a)(-a)$

$=4a^2+40a+36=0$

$\Leftrightarrow a^2+10a+9=0$

$\therefore \underline{a=-9}$ $(\because a\neq -1)$

$\boxed{11}$



$\overrightarrow{OC}=\begin{pmatrix} t \\ 1-t \end{pmatrix}$ $\overrightarrow{AD}=\begin{pmatrix} -1 \\ 1-t \end{pmatrix}$

$\downarrow$

$\overrightarrow{OC}\cdot\overrightarrow{AD}=|\overrightarrow{OC}||\overrightarrow{AD}|\cos\theta$

$\Leftrightarrow -t+(1-t)^2=\sqrt{2t^2-2t+1}\sqrt{t^2-2t+2}\cos\theta$

$\Leftrightarrow \cos\theta=\dfrac{t^2-3t+1}{\sqrt{2t^2-2t+1}\sqrt{t^2-2t+2}}$

$\downarrow$ $\cos^2\theta<\dfrac{1}{2}$

$\dfrac{(t^2-3t+1)^2}{(2t^2-2t+1)(t^2-2t+2)}<\dfrac{1}{2}$

$\Leftrightarrow 2(t^4+9t^2+1-6t^3-6t+2t^2)$

$<2t^4-4t^3+4t^2-2t^3+4t^2-4t$

$+t^2-2t+2$

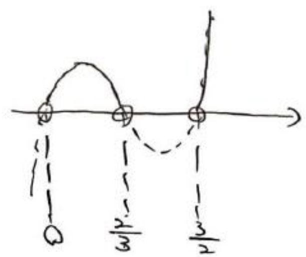
$\Leftrightarrow -12t^3+22t^2-12t+2$

$<-6t^3+9t^2-6t+2$

$\Leftrightarrow 0<6t^3-13t^2+6t$

$\Leftrightarrow 0<t(6t^2-13t+6)$

$\Leftrightarrow 0<t(2t-3)(3t-2)$



$\therefore \underline{0<t<\dfrac{2}{3}}$ $(\because 0<t<1)$

$\boxed{12}$

正の整数解をnとすると

$n^3-20n^2+(100-a)n+8a-23=0$

$\Leftrightarrow (8-n)a=-n^3+20n^2-100n+23$

$\Leftrightarrow a=\dfrac{n^3-20n^2+100n-23}{n-8}$

$$\begin{array}{r|rrrr} & 1 & -12 & 4 & \\ \hline 1\ -8 & 1 & -20 & 100 & -23 \\ & 1 & -8 & & \\ \hline & & -12 & 100 & \\ & & -12 & 96 & \\ \hline & & & 4 & -23 \\ & & & 4 & -32 \\ \hline & & & & 9 \end{array}$$

$a=n^2-12n+4+\dfrac{9}{n-8}$

$=(n-6)^2-32+\dfrac{9}{n-8}$

$n=17$のとき $a=90$

$n=11$ 〃 $a=-4$ … NG

⋮

$\therefore \underline{a=90}$

$\boxed{13}$

$n\cdot a_1+(n-1)a_2+\cdots+1\cdot a_n=\dfrac{n-4}{10}+\dfrac{1}{n+5}$

$-)\ (n-1)a_1+(n-2)a_2+\cdots+1a_{n-1}=\dfrac{n-5}{10}+\dfrac{1}{n+4}$

$a_1+a_2+\cdots+a_n=\dfrac{1}{10}+\dfrac{1}{n+5}-\dfrac{1}{n+4}$

$\therefore \underline{\lim_{n\to\infty}(a_1+a_2+\cdots+a_n)=\dfrac{1}{10}}$

与式に関して

$\sum_{k=1}^{n}(n+1-k)a_k=\dfrac{n-4}{10}+\dfrac{2}{n+5}$

$\Leftrightarrow (n+1)\sum_{k=1}^{n}a_k-\sum_{k=1}^{n}ka_k=$ 〃

$\therefore \sum_{k=1}^{n}ka_k=(n+1)\sum_{k=1}^{n}a_k-\dfrac{n-4}{10}-\dfrac{2}{n+5}$

$\sum_{k=1}^{n}(3k-1)a_k$

$=3\sum_{k=1}^{n}ka_k-\sum_{k=1}^{n}a_k$

$=(3n+2)\sum_{k=1}^{n}a_k-\dfrac{3n-12}{10}-\dfrac{6}{n+5}$

$=(3n+2)\left(\dfrac{1}{10}+\dfrac{1}{n+5}-\dfrac{1}{n+4}\right)$

$-\dfrac{3n-12}{10}-\dfrac{6}{n+5}$

$=\dfrac{14}{10}+\dfrac{3n+2}{n+5}-\dfrac{3n+2}{n+4}-\dfrac{6}{n+5}$

$\therefore \lim_{n\to\infty}\sum_{k=1}^{n}(3k-1)a_k$

$=\underline{\dfrac{14}{10}=\dfrac{7}{5}}$

$\boxed{14}$

$y=\pm\sqrt{(x-1)^2(2x-x^2)}$

$=\pm|x-1|\sqrt{2x-x^2}$

$f(x)=|x-1|\sqrt{\underbrace{2x-x^2}_{\geqq 0}}$ とおく

$\downarrow$

$0 \leqq x \leqq 2$

$f(x)=0 \Longleftrightarrow x=0,1,2$

求める面積をAとおくと

$\frac{A}{2}$

$=\int_0^1 (1-x)\sqrt{1-(1-x)^2}\,dx$

$+\int_1^2 (x-1)\sqrt{1-(x-1)^2}\,dx$

$\begin{cases} 1-x=s \\ x-1=t \end{cases}$

$=\int_1^0 s\sqrt{1-s^2}(-ds)$

$+\int_0^1 t\sqrt{1-t^2}\,dt$

$=\int_0^1 s\sqrt{1-s^2}\,ds+\int_0^1 t\sqrt{1-t^2}\,dt$

$=\int_0^1 2s\sqrt{1-s^2}\,ds$

$=\left[-\frac{2}{3}(1-s^2)^{\frac{3}{2}}\right]_0^1=\frac{2}{3}$

$\therefore A=\frac{4}{3}$

$\boxed{15}$

S_{xy}

$=\overline{xy}-\overline{x}\cdot\overline{y}$

$=5-XY$

S_x^2

$=\overline{x^2}-(\overline{x})^2$

$=5-X^2$

S_y^2

$=9-Y^2$

以上より

$r=\frac{5-XY}{\sqrt{5-X^2}\sqrt{9-Y^2}}$

$\downarrow r^2 \leqq 1$

$\frac{(5-XY)^2}{(5-X^2)(9-Y^2)} \leqq 1$

$\Longleftrightarrow X^2Y^2-10XY+25 \leqq X^2Y^2-9X^2-5Y^2+45$

$\Longleftrightarrow 9X^2-10XY+5Y^2 \leqq 20$

$\Longleftrightarrow 4X^2+5(X-Y)^2 \leqq 20$

$\therefore \frac{(Y-X)^2}{4}+\frac{X^2}{5} \leqq 1$

$X+Y=k$ とおくと

$Y=-X+k$ これを上に代入.

$4X^2+5(2X-k)^2=20$

$\Longleftrightarrow 24X^2-20kX+5k^2-20=0$

$\frac{D}{4}=100k^2-24(5k^2-20)$

$=-20k^2+480 \geqq 0$

$\Longleftrightarrow 24 \geqq k^2$

$\therefore 0 \leqq |k| \leqq 2\sqrt{6}$