2016 帝京(医) 赤本での数学②

[1]

(1) $1-x^6$倍

$12x^2$

$=(1-x+x^2)(1-x)a+(1+x+x^2)(1+x)b$
$\quad+(1-x)(1+x)(cx+d)$

$=(-x^3+2x^2-2x+1)a$
$\quad+(x^3+2x^2+2x+1)b+(1-x^2)(cx+d)$

$=(b-a-c)x^3+(2a+2b-d)x^2$
$\quad+(-2a+2b+c)x+a+b+d$

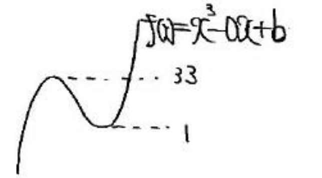
$$\begin{cases} b-a-c=0 \\ 2a+2b-d=12 \\ -2a+2b+c=0 \\ a+b+d=0 \end{cases}$$

$d=-4$

$3b-3a=0 \iff b=a$

$\therefore\ a=b=\underline{2}$, $d=\underline{-4}$

(2)



$f'(x)=3x^2-a$
$\quad=3\left(x^2-\frac{a}{3}\right)$

$f\left(-\sqrt{\frac{a}{3}}\right)=-\sqrt{\frac{a}{3}}\frac{a}{3}+a\sqrt{\frac{a}{3}}+b$
$\quad=\frac{2a}{3}\sqrt{\frac{a}{3}}+b=33$

$f\left(\sqrt{\frac{a}{3}}\right)=-\frac{2a}{3}\sqrt{\frac{a}{3}}+b=1$

$\therefore\ b=\underline{17}$

$\frac{2a}{3}\sqrt{\frac{a}{3}}=16$

$\iff a\sqrt{\frac{a}{3}}=24$

$\downarrow$ 2乗

$\frac{a^3}{3}=24\cdot24$

$\iff a^3=3^3\cdot2^6 \quad \therefore\ a=\underline{12}$

逆の確認は省略.

(3) $\overrightarrow{BC}=\begin{pmatrix}-2\\-2\\4\end{pmatrix}$

直線BC: $\begin{cases} x=3-2t \\ y=4-2t \\ z=4t \end{cases}$

$Q(3-2t,4-2t,4t)$とおく.

$PQ\perp l_1$

$\iff \overrightarrow{PQ}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}=0$

$\iff 3-2t-P_1=0$

$PQ\perp l_2$

$\iff \overrightarrow{PQ}\cdot\overrightarrow{BC}=0$

$\iff -(3-2t-P_1)-(4-2t)+2(4t)=0$

$\iff 12t-7+P_1=0$

$\therefore\ 10t-4=0 \quad t=\frac{2}{5}$

$P_1=3-2\cdot\frac{2}{5}=\underline{\frac{11}{5}}$

$P_2=4-2\cdot\frac{2}{5}=\underline{\frac{16}{5}}$

$P_3=4\cdot\frac{2}{5}=\underline{\frac{8}{5}}$

$\overrightarrow{BP}=\begin{pmatrix}-\frac{4}{5}\\-4\\0\end{pmatrix}$

$\triangle PBC$

$=\frac{1}{2}\sqrt{|\overrightarrow{BC}|^2|\overrightarrow{BP}|^2-(\overrightarrow{BC}\cdot\overrightarrow{BP})^2}$

$=\frac{1}{2}\sqrt{24\cdot\frac{26}{25}\cdot16-\left(\frac{48}{5}\right)^2}$

$=\frac{1}{2}\sqrt{\frac{48}{25}(26\cdot8-48)}$

$=\frac{1}{2}\sqrt{\frac{1}{25}\cdot16\cdot3\cdot16\cdot10}$

$=\frac{1}{2}\cdot\frac{16}{5}\cdot\sqrt{30}=\underline{\frac{8\sqrt{30}}{5}}$

[2]

$a=4,\ b=x,\ c=12-x$

(1) 存在条件

$$\begin{cases} 4+x>12-x & x>0 \\ x+12-x>4 & 12-x>0 \\ 12-x+4>x & \end{cases}$$

$\iff \begin{cases} x>4 & x>0 \\ x<8 & x<12 \end{cases}$

$\therefore\ \underline{4<x<8}$

(2)

$$\begin{cases} \frac{c}{\sin C}=2R \iff R=\frac{12-x}{2\sin C} \\ \frac{1}{2}\cdot4x\sin C=\frac{r}{2}\cdot16 \end{cases}$$

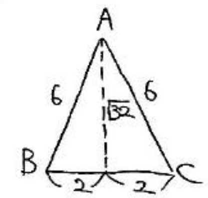
$\iff r=\frac{x}{4}\sin C$

$R\times r$

$=\frac{1}{8}(12-x)x$

$=\frac{1}{8}\{-(x-6)^2+36\}$

$x=\underline{6}$のとき最大. このとき



$S=4\times\sqrt{32}\times\frac{1}{2}=8r$

$\therefore\ r=\underline{\sqrt{2}}$

[3]

対数とる

$\log_2 a_{n+1} = \sqrt{2}\log_2 a_n$

↓一般項

$\log_2 a_n = (\log_2\sqrt{2})(\sqrt{2})^{n-1}$

$= \frac{1}{2}(\log_2 2)\, 2^{\frac{n-1}{2}}$

$= 2^{\frac{n-3}{2}}\log_2 2 = \log_2 2^{2^{\frac{n-3}{2}}}$

$\therefore a_n = 2^{2^{\frac{n-3}{2}}}$

(1)

$a_3 = 2^{2^0} = \underline{2}$

$a_5 = 2^{2^1} = \underline{4}$

(2)

$a_n = 2^{2^{\frac{n-3}{2}}} \geqq 2^{10}\cdot 2^2 = 2^{12}$

$\Leftrightarrow 2^{\frac{n-3}{2}} \geqq 12$

$\Leftrightarrow (\sqrt{2})^{n-3} \geqq 12 > 8\sqrt{2} = (\sqrt{2})^7$

$\therefore n-3 \geqq 8$

$\therefore \min n = \underline{11}$

(3)

$a_n = 2^{2^{\frac{n-3}{2}}} \leqq 2^{1000}$

$\Leftrightarrow 2^{\frac{n-3}{2}} \leqq 1000 < 1024$

$\therefore \frac{n-3}{2} \leqq 9.5$

$\max n = \underline{22}$

[4]

$x=5$ と円②の交点は

$25 + y^2 - 50 - 5y - 125 = 0$

$\Leftrightarrow y^2 - 5y - 150 = 0$

$y = -10, 15$

$\therefore B(\underline{5, 15})$

$T(x_T, y_T)$ とおくと

$l: x_T x + y_T y = 25$

↓ (5,15) とる

$x_T + 3y_T = 5$

また $x_T^2 + y_T^2 = 25$ より

$(5-3y_T)^2 + y_T^2 = 25$

$\Leftrightarrow 10y_T^2 - 30y_T = 0$

$\therefore y_T = 3 \quad (\because y_T \neq 0)$

$\therefore x_T = \underline{-4}$

$l: -4x + 3y = 25$

$\Leftrightarrow y = \frac{4}{3}x + \frac{25}{3}$

②と連立

$x^2 - 10x + \frac{4x+25}{3}\left(\frac{4x+25}{3} - 5\right) - 125 = 0$

$\Leftrightarrow 9x^2 - 90x + (4x+25)(4x+10) - 1125 = 0$

$\Leftrightarrow 25x^2 + 50x + 250 - 1125$

$\Leftrightarrow x^2 + 2x - 35 = 0$

$\therefore x = 5, -7$

$\therefore C(\underline{-7, -1})$

$m: y = \alpha(x+7) - 1$ とおくと

原点との距離が5より

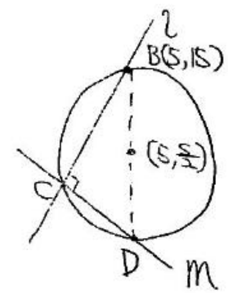
$\frac{|7\alpha - 1|}{\sqrt{\alpha^2+1}} = 5$ ↩ 整理して2乗

$49\alpha^2 - 14\alpha + 1 = 25\alpha^2 + 25$

$\Leftrightarrow 12\alpha^2 - 7\alpha - 12 = 0$

$\Leftrightarrow (3\alpha - 4)(4\alpha + 3) = 0$

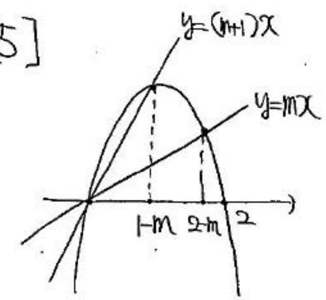
$\therefore \alpha = -\frac{3}{4} \quad (\because m \neq l)$



$l \perp m$ より BDは円②の直径となる.

Dのx座標は図より $x = \underline{5}$

[5]



(1)

$T(m)$

$= \int_0^{1-m}\{(m+1)x - mx\}dx + \int_{1-m}^{2-m}(-x^2+2x-mx)dx$

$= \left[\frac{1}{2}x^2\right]_0^{1-m} + \left[-\frac{x^3}{3} + \frac{2-m}{2}x^2\right]_{1-m}^{2-m}$

$= \frac{1}{2}(1-m)^2 + \frac{1}{6}(2-m)^3 + \frac{1}{3}(1-m)^3 - \frac{2-m}{2}(1-m)^2$

$= \frac{m-1}{2}(1-m)^2 + \frac{1}{6}(2-m)^3 + \frac{1}{3}(1-m)^3$

$= \frac{1}{6}(2-m)^3 - \frac{1}{6}(1-m)^3$

$= \frac{1}{6}\{(2-m)^2 + (2-m)(1-m) + (1-m)^2\}$

$= \frac{1}{6}\underline{(3m^2 - 9m + 7)}$

(2)

$S = \frac{1}{6}(2-0)^3 = \frac{4}{3}$ より

$T(m) = \frac{1}{2}S$

$\Leftrightarrow 3m^2 - 9m + 7 = 4$

$\Leftrightarrow m^2 - 3m + 1 = 0$

$\therefore m = \underline{\frac{3-\sqrt{5}}{2}} \quad (\because 0 \leqq m < 1)$

[6]

(1)

組	x_1	x_2	x_3	
和15	3	6	6	3 通り
	4	5	6	3!
	5	5	5	1
和16	4	6	6	3
	5	5	6	3
和17	5	6	6	3
和18	6	6	6	1

$P(x_1+x_2+x_3 \geqq 15)$

$= \dfrac{3+6+1+3+3+3+1}{216} = \dfrac{5}{54}$ //

(2)

$P(y_2 \geqq 5)$

$= P(y_2=5, y_3=5) + P(y_2=5, y_3=6)$

$+ P(y_2=6, y_3=6)$

$= \dfrac{4\times3+1}{216} + \dfrac{4\times3!+3}{216}$

$+ \dfrac{5\times3+1}{216}$

$= \dfrac{13+27+16}{216} = \dfrac{56}{216}$

$= \dfrac{7}{27}$ //

(3)

$P(y_3 - y_1 \leqq 2)$

$= P(y_3-y_1=0) + P(y_3-y_1=1)$

$+ P(y_3-y_1=2)$

$= \dfrac{6}{216}$

$+ P(y_3-y_1=1 \cap y_1=y_2)$

$+ P(y_3-y_1=1 \cap y_2=y_3)$

$+ P(y_3-y_1=2 \cap y_1=y_2)$

$+ P(y_3-y_1=2 \cap y_2=y_3)$

$+ P(y_3-y_1=2 \cap y_1 \neq y_2 \neq y_3)$

$= \dfrac{1}{36} + \dfrac{5\times3}{216} + \dfrac{5\times3}{216}$

$+ \dfrac{4\times3}{216} + \dfrac{4\times3}{216} + \dfrac{4\times3!}{216}$

$= \dfrac{1}{36} + \dfrac{30+48}{216}$

$= \dfrac{1}{36} + \dfrac{5+8}{36} = \dfrac{7}{18}$ //

(4)

$y_3 - y_1 \leqq 2 \Rightarrow (x_2-x_1)^2+(x_3-x_2)^2+(x_1-x_3)^2 \leqq 9$ …①

は明らか。逆を示す。

$x_1=y_1, x_3=y_3$ としても一般性を失わない。

(i) $|x_1-x_3| \geqq 4$ は①を満たさない。

(ii) $|x_1-x_3|=3$ のとき

$(x_2-x_1)^2+(x_3-x_2)^2 \leqq 0$

これを満たすのは $x_1=x_2=x_3$ になるので不適。

(iii) $|x_1-x_3| \leqq 2$ のとき

$|x_2-x_1| \leqq 1$ かつ $|x_3-x_2| \leqq 2$

または

$|x_2-x_1| \leqq 2$ かつ $|x_3-x_1| \leqq 1$

となるので①を満たす

$\therefore y_3 - y_1 \leqq 2 \Leftarrow (x_2-x_1)^2+(x_3-x_2)^2+(x_1-x_3)^2 \leqq 9$

以上より (3)と(4)の事象は同値。

$P((x_2-x_1)^2+(x_3-x_2)^2+(x_1-x_3)^2 \leqq 9) = \dfrac{7}{18}$ //