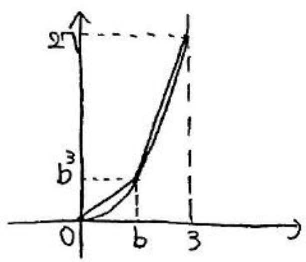
2016 帝京(医) 赤本での数学①

[1]

(1) $\dfrac{3^3-0^3}{3-0}=3a^2$

$\therefore a=\sqrt{3}$ (∵第1象限)



S_1+S_2

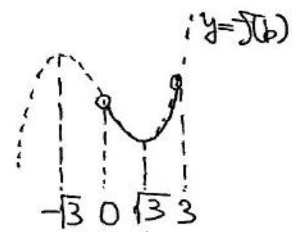
$=b\cdot b^3\frac{1}{2}+(b^3+27)(3-b)\frac{1}{2}-\int_0^3 x^3dx$

$=\frac{1}{2}(3b^3-27b+81)-\frac{81}{4}$

$=\frac{1}{4}(6b^3-54b+81)$

$=\frac{3}{4}\underbrace{(2b^3-18b+27)}_{f(b)}\quad(0<b<3)$

$f'(b)=6b^2-18=6(b^2-3)$



$b=\sqrt{3}$ のとき S_1+S_2 最小.

このとき

$S_1=\frac{9}{2}-\int_0^{\sqrt{3}}x^3dx$

$=\frac{9}{4}$ //

(2)

$t\vec{p}-\vec{q}=\begin{pmatrix}5t-2\\t-4\end{pmatrix}$

$|t\vec{p}-\vec{q}|$

$=\sqrt{25t^2-20t+4+t^2-8t+16}$

$=\sqrt{26t^2-28t+20}$

$t_1=\frac{7}{13}$ //

$t\vec{q}-\vec{p}=\begin{pmatrix}2t-5\\4t-1\end{pmatrix}$

$|t\vec{q}-\vec{p}|$

$=\sqrt{4t^2-20t+25+16t^2-8t+1}$

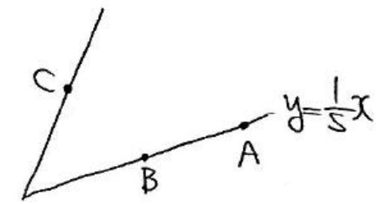
$=\sqrt{20t^2-28t+26}$

$t_2=\frac{7}{10}$ //

$A(5,1),\ B(\frac{35}{13},\frac{7}{13}),\ C(\frac{7}{5},\frac{14}{5})$

△ABCの外接円の中心を

(x,y) とすると



中心はA、Cまでの距離が等しいので

$(x-5)^2+(y-1)^2=(x-\frac{7}{5})^2+(y-\frac{14}{5})^2$

$\Leftrightarrow -10x+25-2y+1=-\frac{14}{5}x+\frac{49}{25}-\frac{28}{5}y+\frac{196}{25}$

$\Leftrightarrow -250x+625-50y+25=-70x-140y+245$

$\Leftrightarrow 90y=180x+245-650$

$\Leftrightarrow y=2x-\frac{9}{2}$

ABの垂直二等分線は

$y=-5(x-\frac{50}{13})+\frac{10}{13}$

$=-5x+\frac{260}{13}=-5x+20$

連立

$2x-\frac{9}{2}=-5x+20$

$\Leftrightarrow 7x=\frac{49}{2}$

$\therefore x=\frac{7}{2}\quad y=\frac{5}{2}$

$R=\sqrt{(5-\frac{7}{2})^2+(1-\frac{5}{2})^2}$

$=\sqrt{\frac{9}{4}+\frac{9}{4}}=\frac{3\sqrt{2}}{2}$ //

(3)

(i) P(3つの積が素数)

$=$ P(2つが1, 1つが素数)

$=\frac{3\cdot 3}{216}$

$=\frac{1}{24}$ //

(ii) P(最大の目が素数)

$=$ P(最大が5) $+$ P(最大が3) $+$ P(最大が2)

$=$ P(すべて5以下) $-$ P(すべて4以下) $+$ P(すべて3以下) $-$ P(すべて2以下) $+$ P(すべて2以下) $-$ P(すべて1以下)

$=\frac{125}{216}-\frac{64}{216}+\frac{27}{216}-\frac{8}{216}+\frac{8}{216}-\frac{1}{216}$

$=\frac{87}{216}$

$=\frac{29}{72}$ //

[2]

(1) 解は $\log_b 2, \frac{1}{2}\log_b 2, \frac{1}{k}\log_b 2$

$\boxed{\text{解と係数}}$

$$\begin{cases} \left(\frac{3}{2}+\frac{1}{k}\right)\log_b 2 = 0 \quad \cdots ① \\ \frac{1}{2}(\log_b 2)^2 + \frac{1}{2k}(\log_b 2)^2 + \frac{1}{k}(\log_b 2)^2 = p \quad \cdots ② \\ \frac{1}{2k}(\log_b 2)^3 = -q \quad \cdots ③ \end{cases}$$

①より

$$\frac{3}{2}+\frac{1}{k}=0 \quad \therefore k=-\frac{2}{3}$$

(2) ②,③を更新

$$\begin{cases} \left(\frac{1}{2}-\frac{3}{4}-\frac{3}{2}\right)(\log_b 2)^2 = p \\ -\frac{3}{4}(\log_b 2)^3 = -q \end{cases}$$

$$\Leftrightarrow \begin{cases} (\log_b 2)^2 = -\frac{4}{7}p \\ (\log_b 2)^3 = \frac{4q}{3} \end{cases}$$

$$\therefore -\frac{64}{7^3}p^3 = \frac{16}{9}q^2$$

$$\Leftrightarrow 64q^3 = \frac{16}{9}q^2$$

$$\therefore q = \frac{1}{36} \quad //$$

$$(\log_b 2)^3 = \frac{1}{27}$$

$$\log_b 2 = \frac{1}{3}$$

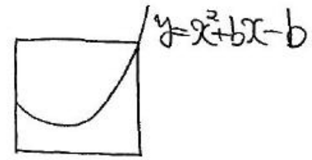
$$\therefore b = 8 \quad //$$

解は順に $\frac{1}{3}, \frac{1}{6}, -\frac{1}{2}$

より $\min x = -\frac{1}{2}$ //

[3]

(1) 放物線がBを通る → $b+c=0$



$$\int_0^1 (x^2+bx-b)\,dx$$

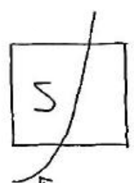
$$=\frac{1}{3}+\frac{b}{2}-b=\frac{1}{2} \quad \therefore b=-\frac{1}{3}$$

$$y=x^2-\frac{1}{3}x+\frac{1}{3}$$

$$=\left(x-\frac{1}{6}\right)^2-\frac{1}{36}+\frac{12}{36}$$

$$\therefore p=\frac{1}{6} \quad q=\frac{11}{36} \quad //$$

(2)



$$S=\int_0^1 \sqrt{y-r}\,dy$$

$$=\left[\frac{2}{3}(y-r)^{\frac{3}{2}}\right]_0^1$$

$$=\frac{2}{3}(1-r)^{\frac{3}{2}}=\frac{1}{2}$$

$$\Leftrightarrow (1-r)^{\frac{3}{2}}=\frac{3}{4}$$

$$\Leftrightarrow 1-r=\left(\frac{3}{4}\right)^{\frac{2}{3}}$$

$$\therefore r=1-\sqrt[3]{\frac{9}{16}}$$

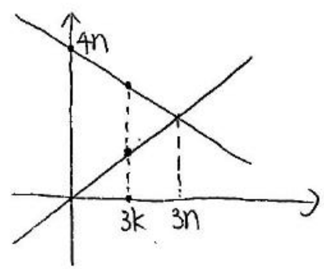
BCとの交点

$$x^2=1-r \quad \therefore x=\sqrt[3]{\frac{3}{4}}$$

$$\left(\sqrt[3]{\frac{3}{4}}, 1\right) \ //, \quad r=1-\sqrt[3]{\frac{9}{16}} \ //$$

[4]

$$\begin{cases} y \geq \frac{2}{3}x \\ y \leq -\frac{2}{3}x+4n \end{cases}$$



(1)(2)

(i) $x=3k$ $(k\geq 0)$ での格子点は

$$-2k+4n-2k+1$$

$$=-4k+4n+1$$

(ii) $x=3k+1$ $(k\geq 0)$ での格子点は

$$-4k+4n+1-2$$

$$=-4k+4n-1$$

(iii) $x=3k+2$ $(k\geq 0)$ での格子点は

$$-4k+4n-1-2$$

$$=-4k+4n-3$$

a_n

$$=\sum_{k=0}^{n}(-4k+4n+1)$$

$$+\sum_{k=0}^{n-1}(-4k+4n-1)$$

$$+\sum_{k=0}^{n-1}(-4k+4n-3)$$

$$=\sum_{k=1}^{n}(-4k+4n+1)+4n+1$$

$$+\sum_{k=1}^{n-1}(-8k+8n-4)+4n-1+4n-3$$

$$=-4\cdot\frac{1}{2}n(n+1)+(4n+1)n-8\cdot\frac{1}{2}(n-1)n+4(2n-1)(n-1)+12n-3$$

$$=-2n^2-2n+4n^2+n-4n^2+4n+4(2n^2-3n+1)+12n-3$$

$$=6n^2+3n+1 \quad //$$

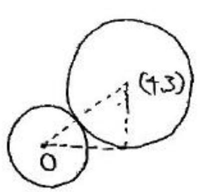
※赤本の解答も見いなと思います.

(1) $a_1=10$ //

[5] $x^2+y^2=4$

$(x-4)^2+(y-3)^2=25-k$

(1)



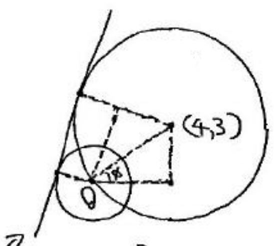
交わる： $3 \leqq \sqrt{25-k} \leqq 7$

$\Leftrightarrow 9 \leqq 25-k \leqq 49$

$\Leftrightarrow -24 \leqq k \leqq 16$

$\min k = \underline{-24}$ //

(2) 円②の半径が5のとき



$\tan\varphi = \frac{3}{4}$

$(傾き) = \tan 2\varphi = \frac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{24}{7}$

$y=\frac{24}{7}x+b$ と原点の距離が2より

$$\frac{|b|}{\sqrt{\left(\frac{24}{7}\right)^2+1}} = 2$$

$\Leftrightarrow |b| = 2\cdot\frac{\sqrt{24^2+7^2}}{7} = \frac{50}{7}$

求める直線は $y=\frac{24}{7}x+\frac{50}{7}$ //

(3) 円②の半径が5 $\Leftrightarrow k=0$

円①②の共有点を通る円は

$x^2+y^2-8x-6y+t(x^2+y^2-4)=0$

$\downarrow$ (1,1)を通る

$-12-2t=0 \quad \therefore t=-6$

$\downarrow$

$-5x^2-5y^2-8x-6y+24=0$

$\Leftrightarrow x^2+y^2+\frac{8}{5}x+\frac{6}{5}y-\frac{24}{5}=0$

$\Leftrightarrow \left(x+\frac{4}{5}\right)^2+\left(y+\frac{3}{5}\right)^2=\frac{24}{5}+\frac{16}{25}+\frac{9}{25}$

$=\frac{145}{25}$

$=\frac{29}{5}$

円の面積

$\underline{\frac{29}{5}\pi}$ //

[6]

(1)

$P(\text{Aが赤2コで勝つ})$

$=P(\text{Aが赤2コ, Bが赤1コ})$

$+P(\text{Aが赤2コ, Bが赤なし})$

$=\frac{{}_6C_2}{{}_{10}C_2}\cdot\frac{{}_4C_1\cdot{}_4C_2}{{}_8C_3}+\frac{{}_6C_2}{{}_{10}C_2}\cdot\frac{{}_4C_3}{{}_8C_3}$

$=\frac{30}{90}\cdot\frac{24+4}{56}=\frac{1}{3}\cdot\frac{4}{8}$

$=\underline{\frac{1}{6}}$ //

(2) $P(\text{A勝つ})$

$=(1)+P(\text{Aが赤1コ, B赤なし})$

$=\frac{1}{6}+\frac{6\cdot4}{{}_{10}C_2}\cdot\frac{{}_3C_3}{{}_8C_3}$

$=\frac{1}{6}+\frac{24}{45}\cdot\frac{1}{56}$

$=\frac{1}{6}+\frac{1}{15}\cdot\frac{1}{7}=\underline{\frac{37}{210}}$ //

(3)

$P(\text{Bが赤3コで勝つ})$

$=P(\text{A赤なし, B赤3コ})$

$+P(\text{A赤1コ, B赤3コ})$

$+P(\text{A赤2コ, B赤3コ})$

$=\frac{{}_4C_2}{{}_{10}C_2}\cdot\frac{{}_6C_3}{{}_8C_3}+\frac{6\cdot4}{{}_{10}C_2}\cdot\frac{{}_5C_3}{{}_8C_3}$

$+\frac{{}_6C_2}{{}_{10}C_2}\cdot\frac{{}_4C_3}{{}_8C_3}$

$=\frac{6\cdot20+24\cdot10+15\cdot4}{45\cdot56}$

$=\frac{6\cdot5+6\cdot10+15}{45\cdot14}$

$=\frac{2+4+1}{3\cdot14}=\underline{\frac{1}{6}}$ //

(4) $P(\text{B勝つ})$

$=1-P(\text{A勝つ})-P(\text{引き分け})$

$=\frac{173}{210}-P(\text{A赤1コ, B赤1コ})$

$-P(\text{A赤2コ, B赤2コ})$

$=\frac{173}{210}-\frac{6\cdot4}{{}_{10}C_2}\cdot\frac{5\cdot{}_3C_2}{{}_8C_3}$

$-\frac{{}_6C_2}{{}_{10}C_2}\cdot\frac{{}_4C_2\cdot4}{{}_8C_3}$

$=\frac{173}{210}-\frac{24\cdot15+15\cdot24}{45\cdot56}$

$=\frac{173}{210}-\frac{2}{7}$

$=\underline{\frac{113}{210}}$ //