2016 昭和大医 I期

1

(1)

$${}_{10}C_8\left(\frac{1}{10}\right)^8\left(\frac{9}{10}\right)^2$$

$$= 45\cdot\frac{81}{10^{10}}$$

$$= \underline{\frac{3645}{10^{10}}}$$

(2)

$$P_n = {}_nC_8\left(\frac{1}{n}\right)^8\left(\frac{n-1}{n}\right)^{n-8}$$

$$= \underline{\frac{n!}{(n-8)!\,8!}\cdot\frac{(n-1)^{n-8}}{n^n}}$$

(3)

$$\lim_{n\to\infty}P_n$$

$$= \lim_{n\to\infty}\frac{n(n-1)\cdots(n-7)}{8!}\left(1-\frac{1}{n}\right)^{n-8}\frac{1}{n^8}$$

$$= \lim_{n\to\infty}\frac{1}{8!}\cdot 1\cdot\left(1-\frac{1}{n}\right)\cdots\left(1-\frac{7}{n}\right)\left\{\left(1+\frac{1}{-n}\right)^{-n}\right\}^{-1}\left(1-\frac{1}{n}\right)^{-8}$$

$$= \underline{\frac{1}{8!\,e}}$$

2

(1)

(1-1)

$$\vec{a}\cdot\vec{d} = \vec{a}\cdot\vec{b} - \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}|\vec{a}|^2 = \underline{0}$$

(1-2)

$$\vec{e}\cdot\vec{d}$$

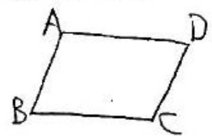
$$= \vec{c}\cdot\vec{d} - \frac{\vec{a}\cdot\vec{c}}{|\vec{c}|^2}\underbrace{\vec{a}\cdot\vec{d}}_{0} - \frac{\vec{d}\cdot\vec{c}}{|\vec{d}|^2}|\vec{d}|^2$$

$$= \underline{0}$$

(2)

$A(1,1), B(3,-7), C(-1,-2)$

$D(x,y)$ とすると



$$\overrightarrow{AD} = \overrightarrow{BC}$$

$$\begin{pmatrix} x-1 \\ y-1 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

$\therefore D(-3,6)$

$\therefore D : \underline{-3+6i}$

(3)

(3-1) 問題文にある数列を S_n とする．

$\{a_n\}: 1, 10, 100, \cdots, 10^{n-1}, \cdots$

$$S_k = \frac{1-10^k}{1-10} = \underline{\frac{10^k-1}{9}}$$

(3-2)

(和)

$$= \sum_{k=1}^{n}\left(\frac{1}{9}10^k - \frac{1}{9}\right)$$

$$= \frac{1}{9}\cdot\frac{10-10^{n+1}}{1-10} - \frac{1}{9}n$$

$$= \underline{\frac{10^{n+1}-10}{81} - \frac{n}{9}}$$

3

(1)

(1-1)

$$\begin{array}{r} 9\,\underline{)\,2016} \\ 8\,\underline{)\,224} \\ 7\,\underline{)\,28} \\ 4 \end{array}$$

$2016 = 2^5\cdot 3^2\cdot 7$

約数：$6\times 3\times 2 = \underline{36}$ コ

(1-2)

$$(1+2+\cdots+2^5)(1+3+9)(1+7)$$

$$= \frac{1-2^6}{1-2}\cdot 13\cdot 8 = \underline{6552}$$

(2)

(与式)

$$= \log_3\left(a+\frac{3}{b}\right)\left(b+\frac{3}{a}\right)$$

$$= \log_3\left(ab+\frac{9}{ab}+6\right)$$

$$\geq \log_3\left(2\sqrt{ab\cdot\frac{9}{ab}}+6\right)$$

$$= \log_3 12 = \underline{2\log_3 2+1}$$

等号成立は $ab = \frac{9}{ab} \Leftrightarrow ab = 3$

(3)

$$(s+c)^2 = 1+2sc = \frac{4}{9}$$

$$\therefore sc = -\frac{5}{18}$$

$$(c-s)^2 = 1-2sc = \frac{14}{9}$$

$$\therefore c-s = \frac{\sqrt{14}}{3}$$

$$c^3 - s^3$$

$$= (c-s)(c^2+cs+s^2)$$

$$= \frac{\sqrt{14}}{3}\cdot\frac{13}{18}$$

$$= \underline{\frac{13\sqrt{14}}{54}}$$

(4)

$$\begin{array}{rl} & \sin(2x+x)=\sin 2x\cos x+\cos 2x\sin x \\ + & \sin(2x-x)=\sin 2x\cos x-\cos 2x\sin x \\ \hline & \sin 3x+\sin x=2\sin 2x\cos x \end{array}$$

より

$$|\sin 3x+\sin 2x+\sin x|$$

$$=|\sin 2x(2\cos x+1)|$$

$$=\begin{cases} \sin 3x+\sin 2x+\sin x & \left(0\leqq x\leqq\frac{\pi}{2},\ \frac{2\pi}{3}\leqq x\leqq\pi\right) \\ -\sin 3x-\sin 2x-\sin x & \left(\frac{\pi}{2}\leqq x\leqq\frac{2\pi}{3}\right) \end{cases}$$

(与式)

$$=\Big[\underbrace{-\frac{1}{3}\cos 3x-\frac{1}{2}\cos 2x-\cos x}_{F(x)}\Big]_0^{\frac{\pi}{2}}$$

$$+\left[\frac{1}{3}\cos 3x+\frac{1}{2}\cos 2x+\cos x\right]_{\frac{\pi}{2}}^{\frac{2\pi}{3}}$$

$$+\left[-\frac{1}{3}\cos 3x-\frac{1}{2}\cos 2x-\cos x\right]_{\frac{2\pi}{3}}^{\pi}$$

$$=F\left(\frac{\pi}{2}\right)-F(0)-F\left(\frac{2\pi}{3}\right)+F\left(\frac{\pi}{2}\right)+F(\pi)-F\left(\frac{2\pi}{3}\right)$$

$$=2\cdot\frac{1}{2}-\frac{11}{6}-2\left(-\frac{1}{3}+\frac{1}{4}+\frac{1}{2}\right)+\left(\frac{1}{3}-\frac{1}{2}+1\right)$$

$$=\frac{17}{6}$$

$\boxed{4}$

(1)

S

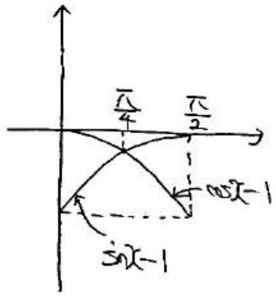
$$=\int_0^{\frac{\pi}{4}}(1-\cos x)dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(1-\sin x)dx$$

$$=\left[x-\sin x\right]_0^{\frac{\pi}{4}}+\left[x+\cos x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$=\frac{\pi}{4}-\frac{\sqrt{2}}{2}+\frac{\pi}{2}-\left(\frac{\pi}{4}+\frac{\sqrt{2}}{2}\right)$$

$$=\frac{\pi}{2}-\sqrt{2}$$

(2) y方向に−1ずらして、x軸まわりの回転体にする。



V

$$=\int_0^{\frac{\pi}{4}}(\cos x-1)^2\pi dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin x-1)^2dx$$

$$=\pi\int_0^{\frac{\pi}{4}}\left(\frac{1+\cos 2x}{2}-2\cos x+1\right)dx$$

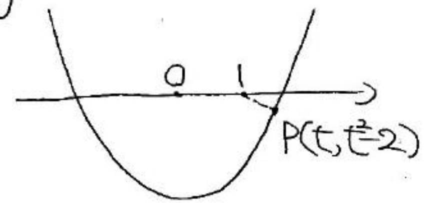
$$+\pi\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\frac{1-\cos 2x}{2}-2\sin x+1\right)dx$$

$$=\pi\left[\frac{3}{2}x+\frac{1}{4}\sin 2x-2\cos x\right]_0^{\frac{\pi}{4}}$$

$$+\pi\left[\frac{3}{2}x-\frac{1}{4}\sin 2x+2\cos x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$=\pi\left(\frac{3}{4}\pi+\frac{1}{2}-2\sqrt{2}\right)$$

(2)



$\text{有}l^2$

$$=(t-1)^2+(t^2-2)^2$$

$$=t^4-3t^2-2t+5=f(t)$$

$$f'(t)=4t^3-6t-2$$

$$=2(2t^3-3t-1)$$

$$=2(t+1)(2t^2-2t-1)$$

$$f'(t)=0\Leftrightarrow t=-1,\ \frac{1\pm\sqrt{3}}{2}$$

t	$\cdots$	-1	$\cdots$	$\frac{1-\sqrt{3}}{2}$	$\cdots$	$\frac{1+\sqrt{3}}{2}$	$\cdots$
$f'(t)$	$-$	0	$+$	0	$-$	0	$+$
$f(t)$	↘		↗		↘		↗

図から明らかに $t=\frac{1+\sqrt{3}}{2}$ のとき最小値をとる。

そのとき

$$t^2-2=\frac{4+2\sqrt{3}}{4}-2$$

$$=\frac{\sqrt{3}}{2}-1$$

$$\therefore P\left(\frac{1+\sqrt{3}}{2},\ \frac{\sqrt{3}-2}{2}\right)$$