2016 埼玉医（前期）

$\boxed{1}$

問1. $n^3+547=m^3$

$\Leftrightarrow \underbrace{547}_{\text{素数}}=\underbrace{(m-n)}_{1}(m^2+mn+n^2)$

$\downarrow$

$n^3+547=(n+1)^3$

$547=3n^2+3n+1$

$182=n^2+n=n(n+1)$

$\therefore \underline{n=-14,\ 13}$ //

問2.

$y=\dfrac{e^x-e^{-x}}{2}$ の逆関数の

$x=\dfrac{e^y-e^{-y}}{2}$

$=\sinh y$ の話.

$1+x^2=1+\sinh^2 y$ $\boxed{\cosh^2 y-\sinh^2 y=1}$

$=\cosh^2 y$

$=\left(\dfrac{e^y+e^{-y}}{2}\right)^2 \cdots \underline{③}$ //

$f(y)=\dfrac{e^y-e^{-y}}{2}=x$

より

$\dfrac{df(y)}{dx}=1 \cdots \underline{⑨}$ //

$x=\dfrac{e^y-e^{-y}}{2}$

$\Leftrightarrow 2x=e^y-e^{-y}$

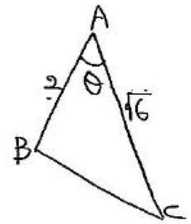
$\Leftrightarrow 0=(e^y)^2-2xe^y-1$

$\Leftrightarrow e^y=x+\sqrt{x^2+1}$

$(\because e^y>0)$

$\therefore y=\log_e(x+\sqrt{x^2+1}) \cdots \underline{⑧}$ //

問3.



$\frac{1}{2}\cdot 2\sqrt{6}\sin\theta=\dfrac{3-\sqrt{3}}{2}$

$\Leftrightarrow \sin\theta=\dfrac{3-\sqrt{3}}{2\sqrt{6}}=\dfrac{\sqrt{6}-\sqrt{2}}{4}$

$\downarrow$

$\theta=15^\circ$

余弦定理

$BC^2=4+6-2\cdot 2\sqrt{6}\cos 15^\circ$

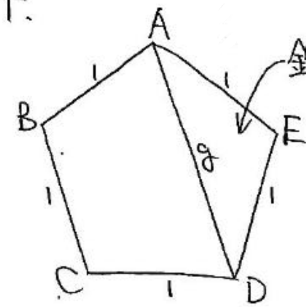
$=10-4\sqrt{6}\cdot\dfrac{\sqrt{6}+\sqrt{2}}{4}$

$=4-2\sqrt{3}$

$\therefore BC=\sqrt{3}-1$

$=\underline{-1+\sqrt{3}}$ //

問4.



$\overrightarrow{AD}=g\overrightarrow{BC}$

$=\underline{\dfrac{1+\sqrt{5}}{2}\overrightarrow{BC}}$ //

$\overrightarrow{CD}=\overrightarrow{CB}+\overrightarrow{BA}+\overrightarrow{AD}$

$=-\vec{b}-\vec{a}+\dfrac{1+\sqrt{5}}{2}\vec{b}$

$=\underline{-\vec{a}+\dfrac{\sqrt{5}-1}{2}\vec{b}}$ //

$\boxed{2}$

問1. (16)の部分和をS_nとおく

$S_n=e^a+2e^{2a}+\cdots+ne^{na}$

$-\big)\ e^aS_n=\qquad e^{2a}+\cdots+(n-1)e^{na}+ne^{(n+1)a}$

$(1-e^a)S_n=e^a+e^{2a}+\cdots+e^{na}-ne^{(n+1)a}$

$=\dfrac{e^a-e^{(n+1)a}}{1-e^a}-ne^{(n+1)a}$

$\therefore S_n=\dfrac{e^a-e^{(n+1)a}}{(1-e^a)^2}-\dfrac{ne^{(n+1)a}}{1-e^a}$

$a<0$ より

$(16)=\lim\limits_{n\to\infty}S_n$

$=\dfrac{e^a}{(1-e^a)^2}$

(17)の部分和をT_nとおく

$T_n=e^a+2^2e^{2a}+\cdots+n^2e^{na}$

$-\big)\ e^aT_n=\qquad e^{2a}+\cdots+(n-1)^2e^{na}+n^2e^{(n+1)a}$

$(1-e^a)T_n=e^a+3e^{2a}+\cdots+(2n-1)e^{na}$
$-n^2e^{(n+1)a}$

$e^a(1-e^a)T_n=\qquad e^{2a}+\cdots+(2n-3)e^{na}$
$+(2n-1)e^{(n+1)a}-n^2e^{(n+2)a}$

$-\big)$

$(e^a-1)^2T_n=e^a+2e^{2a}+\cdots+2e^{na}$
$-(2n-1)e^{(n+1)a}+n^2e^{(n+2)a}$

$=e^a+2\dfrac{e^{2a}-e^{(n+1)a}}{1-e^a}$
$-(2n-1)e^{(n+1)a}+n^2e^{(n+2)a}$

$T_n=\dfrac{e^a}{(1-e^a)^2}+2\dfrac{e^{2a}-e^{(n+1)a}}{(1-e^a)^3}-\dfrac{\sim}{(1-e^a)^2}$

$(17)=\lim\limits_{n\to\infty}T_n$

$=\dfrac{e^a}{(1-e^a)^2}+\dfrac{2e^{2a}}{(1-e^a)^3}$

$=\dfrac{e^a+e^{2a}}{(1-e^a)^3}=\underline{\dfrac{e^a(1+e^a)}{(1-e^a)^3}}$ //

問2

(与式)

$= \sum_{n=1}^{\infty} \left\{ \frac{1}{n(n+3)} - \frac{1}{(n+3)(n+6)} \right\}$

$= \lim_{n \to \infty} \left\{ \left(\frac{1}{1\cdot 4} - \frac{1}{4\cdot 7} \right) + \left(\frac{1}{2\cdot 5} - \frac{1}{5\cdot 8} \right) + \left(\frac{1}{3\cdot 6} - \frac{1}{6\cdot 9} \right) + \left(\frac{1}{4\cdot 7} - \frac{1}{7\cdot 10} \right) + \left(\frac{1}{5\cdot 8} - \frac{1}{8\cdot 11} \right) + \left(\frac{1}{6\cdot 9} - \frac{1}{9\cdot 12} \right) + \cdots \right\}$

$= \frac{1}{1\cdot 4} + \frac{1}{2\cdot 5} + \frac{1}{3\cdot 6}$

$= \frac{45+18+10}{180}$

$= \underline{\frac{73}{180}}$ //

3

問1 $ax^2 = x\log x$

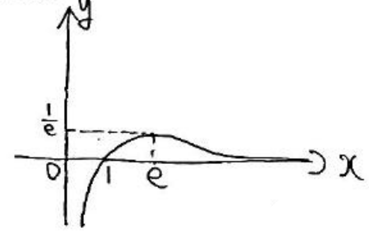
$\iff a = \underbrace{\frac{\log x}{x}}_{f(x)とおく} \quad (x>0)$

$f'(x) = \frac{1-\log x}{x^2}$

x	0	$\cdots$	e	$\cdots$
$f'(x)$	×	+	0	−
$f(x)$	×	↗	$\frac{1}{e}$	↘

$\lim_{x \to +0} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = 0$



$y=f(x)$ と $y=a$ の交点が2つである範囲は図より

$\underline{0 < a < \frac{1}{e}}$ //

問2

$a = \frac{\log p}{p} = \frac{\log q}{q} = \frac{2\log p}{p^2}$

$\frac{1}{p} = \frac{2}{p^2} \quad (\because \log p > 0)$

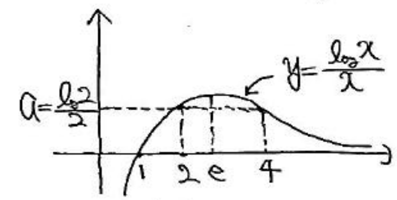
$\therefore p = \underline{2}$ //

$\therefore q = p^2 = \underline{4}$ //

$\therefore a = \underline{\frac{\log 2}{2}}$ //

問3

$2<x<4$ と $a = \frac{\log 2}{2}$ において



図より $\frac{\log x}{x} > a$

$\iff x\log x > ax^2$

より

$S = \int_2^4 (x\log x - ax^2)\,dx$

$= \left[\frac{x^2}{2}\log x - \frac{x^2}{4} - \frac{ax^3}{3} \right]_2^4$

$= 8\log 4 - 4 - \frac{64a}{3} - 2\log 2 + 1 + \frac{8a}{3}$

$= 14\log 2 - 3 - \frac{56}{3}a$

$= \underline{\frac{14}{3}\log 2 - 3}$ //

4

問1

P(第1列と第1行どちらにも白×1、赤×2)

$= P\begin{pmatrix} W & R & R \\ R & & \\ R & & \end{pmatrix} + P\begin{pmatrix} R & \boxed{\ } & \\ \boxed{\ } & & \end{pmatrix}$

WR or RW

W R or R W

$= \frac{2}{3}\left(\frac{1}{3}\right)^4 + 2^2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3$

$= \underline{\frac{2}{27}}$ //

問2

$P_{問1}$(第2列に赤が少なくとも1枚)

$= \frac{P\{問1 \cap (問1-第2列に赤がない)\}}{P(問1)}$

$= \frac{\frac{2}{27} - P\begin{pmatrix} R & W & R \\ \boxed{\ } & W & \\ & W & \end{pmatrix}}{\frac{2}{27}}$

W R or R W

$= 1 - \frac{27}{2}\cdot 2\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^3$

$= \underline{\frac{65}{81}}$ //