2016 日本大(医)

[1]

(1)
$$\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{4}{\sqrt{5}-1}$$
$$=7+4\sqrt{3}+\sqrt{5}+1$$
$$=8+4\underbrace{\sqrt{3}}_{1.73}+\underbrace{\sqrt{5}}_{2.23}$$
$$=\underbrace{17}_{a}+\underbrace{4\sqrt{3}+\sqrt{5}-9}_{b}$$
$\underline{a=17,\ b=4\sqrt{3}+\sqrt{5}-9}$ //

(2)
必要条件 $k\neq0$

$D=(k+1)^2-4k\cdot k$

$=-3k^2+2k+1>0$

$\Leftrightarrow 3k^2-2k-1<0$

$\Leftrightarrow (3k+1)(k-1)<0$

$\therefore \underline{-\frac{1}{3}<k<0}$ または $\underline{0<k<1}$ //

(3)
正弦定理

$$\frac{5}{\sin C}=\frac{2\sqrt{3}}{\sin 30^\circ}=4\sqrt{3}=2R$$
$$\Leftrightarrow \sin C=\frac{5}{4\sqrt{3}}=\underline{\frac{5\sqrt{3}}{12}}\ //$$

$R=\underline{2\sqrt{3}}$ //

(4)
P(4回目で2度目の赤)
$$={}_3C_1\left(\frac{5}{10}\right)^1\left(\frac{5}{10}\right)^2\times\frac{5}{10}$$
$$=\underline{\frac{3}{16}}\ //$$

[2]

(1) 解を $1+2i$, $1-2i$, γ とすると解と係数の関係より

$$\begin{cases} 1+2i+1-2i+\gamma=-a \\ (1+2i)(1-2i)+(1-2i)\gamma+\gamma(1+2i)=-9 \\ (1+2i)(1-2i)\gamma=-b \end{cases}$$

$$\Leftrightarrow \begin{cases} 2+\gamma=-a \\ 5+2\gamma=-9 \\ 4\gamma=-b \end{cases}$$

$\therefore \gamma=-7,\ \underline{a=5,\ b=28}$ //

$\underline{x=-7}$ //

(2)
y
$$=3\frac{1-\cos2\theta}{2}-5\frac{1}{2}\sin2\theta-(1+\cos2\theta)$$
$$=\frac{1}{2}-\frac{5}{2}(\sin2\theta+\cos2\theta)$$
$$=\frac{1}{2}-\frac{5\sqrt{2}}{2}\sin\left(2\theta+\frac{\pi}{4}\right)$$
$$\left(\frac{\pi}{4}\leqq2\theta+\frac{\pi}{4}\leqq\frac{5}{4}\pi\right)$$

$\max y=\underline{3}$ // $\left(\theta=\frac{\pi}{2}\right)$

$\min y=\underline{\frac{1-5\sqrt{2}}{2}}$ // $\left(\theta=\frac{\pi}{8}\right)$

(3)
$$\log_x y+\frac{1}{\log_x y}=2$$
$$\Leftrightarrow (\log_x y)^2-2\log_x y+1=0$$

$\therefore \log_x y=1$

$\therefore y=x$

$\therefore x^2=3^{126}$

$\therefore x=3^{63}$

$\log_{10}x=63\cdot0.4771$

$=30.0573$

$30<\log_{10}x<31$

$\Leftrightarrow 10^{30}<x<10^{31}$

$\therefore \underline{x は31桁}$ //

(4)
$$3\log_3 a_{n+1}=\log_3 a_n$$
$$\Leftrightarrow \log_3 a_{n+1}=\frac{1}{3}\log_3 a_n$$

$$\lim_{n\to\infty}S_n$$
$$=\lim_{n\to\infty}\sum_{k=1}^{n}\log_3 a_k$$
$$=\frac{\log_3 a_1}{1-\frac{1}{3}}$$
$$=\frac{\frac{3}{2}}{\frac{2}{3}}=\underline{\frac{9}{4}}\ //$$

[3]

(1)

$|\overrightarrow{BC}|^2$

$= |\overrightarrow{AC}-\overrightarrow{AB}|^2$

$= |\overrightarrow{AC}|^2 - 2\overrightarrow{AB}\cdot\overrightarrow{AC} + |\overrightarrow{AB}|^2$

$= 2$

$\therefore BC = \sqrt{2}$ //

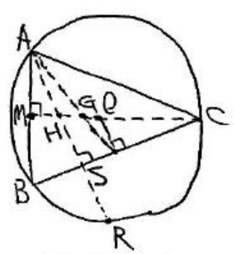
$\triangle ABC$

$= \frac{1}{2}\sqrt{|\overrightarrow{AB}|^2|\overrightarrow{AC}|^2 - (\overrightarrow{AB}\cdot\overrightarrow{AC})^2}$

$= \frac{1}{2}\sqrt{1\cdot 2 - \frac{1}{4}}$

$= \frac{\sqrt{7}}{4}$ //

(2)



余 $\cos C = \frac{2+2-1}{2\sqrt{2}\sqrt{2}} = \frac{3}{4}$

$\sin C = \frac{\sqrt{7}}{4}$

正 $\frac{AB}{\sin C} = 2R \quad \therefore R = \frac{2}{\sqrt{7}}$

$MC = \sqrt{2 - \frac{1}{4}} = \frac{\sqrt{7}}{2}$

$MO = \frac{\sqrt{7}}{2} - \frac{2}{\sqrt{7}}$

$\overrightarrow{AO} = \dfrac{\frac{2}{\sqrt{7}}\overrightarrow{AM} + \left(\frac{\sqrt{7}}{2} - \frac{2}{\sqrt{7}}\right)\overrightarrow{AC}}{\frac{\sqrt{7}}{2} - \frac{2}{\sqrt{7}} + \frac{2}{\sqrt{7}}}$

$= \frac{4}{7}\overrightarrow{AM} + \frac{2}{\sqrt{7}}\left(\frac{\sqrt{7}}{2} - \frac{2}{\sqrt{7}}\right)\overrightarrow{AC}$

$= \frac{2}{7}\overrightarrow{AB} + \frac{3}{7}\overrightarrow{AC}$ //

(3)

図より

$OG : GH = 1 : 2.$

$\overrightarrow{AH} = \overrightarrow{AO} + \overrightarrow{OH}$

$= \overrightarrow{AO} + 3\overrightarrow{OG}$

$= \overrightarrow{AO} + 3(\overrightarrow{AG} - \overrightarrow{AO})$

$= -2\overrightarrow{AO} + 3\overrightarrow{AG}$

$= \left(-\frac{4}{7}+1\right)\overrightarrow{AB} + \left(-\frac{6}{7}+1\right)\overrightarrow{AC}$

$= \frac{3}{7}\overrightarrow{AB} + \frac{1}{7}\overrightarrow{AC}$

$\overrightarrow{OR} = \overrightarrow{AR} - \overrightarrow{AO}$

$= t\overrightarrow{AH} - \overrightarrow{AO}$

$= \left(\frac{3}{7}t - \frac{2}{7}\right)\overrightarrow{AB} + \left(\frac{t}{7} - \frac{3}{7}\right)\overrightarrow{AC}$

$|\overrightarrow{OR}|^2$

$= \frac{1}{49}(3t-2)^2 + 2\frac{1}{49}(3t-2)(t-3)\frac{1}{2}$

$+ \frac{1}{49}(t-3)^2 2 = \frac{4}{7}$

$\Leftrightarrow (3t-2)^2 + (3t-2)(t-3)$

$+ 2(t-3)^2 = 28$

$\Leftrightarrow 14t^2 - 35t = 0$

$t = \frac{5}{2}$

$\therefore \overrightarrow{AR} = t\overrightarrow{AH}$

$= \frac{15}{14}\overrightarrow{AB} + \frac{5}{14}\overrightarrow{AC}$ //

$\overrightarrow{AS} = \frac{3}{4}\overrightarrow{AB} + \frac{1}{4}\overrightarrow{AC}$

$AS : AR = \frac{3}{4} : \frac{15}{14}$

$= 7 : 10$

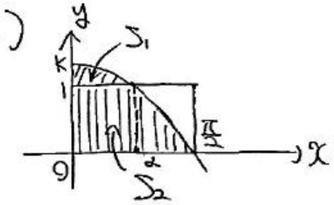
四角形ABRC

$= \triangle ABC \times \frac{10}{7}$

$= \frac{5}{14}\sqrt{7}$ //

[4]

(1)



$k\cos\alpha = 1$ とする. $\left(0 < \alpha < \frac{\pi}{2}\right)$

$S_1 = \int_0^{\alpha}(k\cos x - 1)dx$

$= [k\sin x - x]_0^{\alpha}$

$= k\sin\alpha - \alpha$

$S_2 = \alpha + \int_{\alpha}^{\frac{\pi}{2}} k\cos x\,dx$

$= \alpha + k - k\sin\alpha$

$\lim_{k\to\infty}\frac{kS_2}{S_1}$

$= \lim_{k\to\infty}\frac{\alpha k + k^2 - k^2\sqrt{\sin^2\alpha}}{k\sqrt{\sin^2\alpha} - \alpha}$

$= \lim_{k\to\infty}\frac{\alpha k + k^2 - k^2\sqrt{1-\frac{1}{k^2}}}{k\sqrt{1-\frac{1}{k^2}} - \alpha}$

$= \lim_{k\to\infty}\frac{\alpha k + k^2 - k\sqrt{k^2-1}}{\sqrt{k^2-1} - \alpha}$

$= \lim_{k\to\infty}\left\{\frac{\alpha k}{\sqrt{k^2-1}-\alpha} + \frac{k(k-\sqrt{k^2-1})(k+\sqrt{k^2-1})}{(\sqrt{k^2-1}-\alpha)(k+\sqrt{k^2-1})}\right\}$

$= \lim_{k\to\infty}\left\{\frac{\alpha}{\sqrt{1-\frac{1}{k^2}}-\frac{\alpha}{k}} + \frac{1}{\left(\sqrt{1-\frac{1}{k^2}}-\frac{\alpha}{k}\right)(k+\sqrt{k^2-1})}\right\}$

$\cos\alpha = \dfrac{1}{k}$ より

$k \to \infty$ なら $\cos\alpha \to 0$. つまり $\alpha \to \dfrac{\pi}{2}$.

$$\lim_{k\to\infty} \frac{kS_2}{S_1} = \frac{\pi}{2} \quad /\!/$$

(2)

$k\cos t = 1$

$\cos t = \dfrac{1}{k}$ …①

$\boxed{\text{一応}\quad t = \cos^{-1}\dfrac{1}{k}}$

$S_2 - S_1$

$= t + k - k\sin t - (k\sin t - t)$

$= 2t - 2k\sin t + k$

$= 2f(k) - 2k\dfrac{\sqrt{k^2-1}}{k} + k$

$= \underline{2f(k) - 2\sqrt{k^2-1} + k}\ /\!/$

$\dfrac{d}{dk}(S_2 - S_1)$

$= 2\dfrac{dt}{dk} - (k^2-1)^{-\frac{1}{2}}(2k) + 1$

①をkで微分

$-\sin t\dfrac{dt}{dk} = -\dfrac{1}{k^2}$

$\Leftrightarrow \dfrac{dt}{dk} = \dfrac{1}{k^2\sin t} = \dfrac{1}{k\sqrt{k^2-1}}$

$= \dfrac{2}{k\sqrt{k^2-1}} - \dfrac{2k}{\sqrt{k^2-1}} + 1$

$= 1 - \dfrac{2(k^2-1)}{k\sqrt{k^2-1}}$

$= \dfrac{k - 2\sqrt{k^2-1}}{k} \times \dfrac{k + 2\sqrt{k^2-1}}{k + 2\sqrt{k^2-1}}$

$= \dfrac{k^2 - 4(k^2-1)}{k(k + 2\sqrt{k^2-1})}$

$= \dfrac{4 - 3k^2}{k(k + 2\sqrt{k^2-1})}$

k	1 … $\frac{2}{\sqrt{3}}$ …
$\frac{d}{dk}(S_2-S_1)$	× + 0 −
$S_2 - S_1$	× ↗ ↘

$\underline{k = \dfrac{2}{\sqrt{3}}\text{のとき}}\ /\!/$

最大値 $2f\left(\dfrac{2}{\sqrt{3}}\right) - 2\sqrt{\dfrac{1}{3}} + \dfrac{2}{\sqrt{3}}$

$= 2\dfrac{\pi}{6} - \dfrac{2}{\sqrt{3}} + \dfrac{2}{\sqrt{3}} = \underline{\dfrac{\pi}{3}}\ /\!/$

[5]

(1) $\dfrac{a}{e}x^2 = 2a\log x$

$\Leftrightarrow x^2 = 2e\log x$

$\Leftrightarrow \underbrace{x^2 - 2e\log x = 0}_{f(x)}$

$f'(x) = 2x - \dfrac{2e}{x} = \dfrac{2(x^2 - e)}{x}$

x	0 … $\sqrt{e}$ …
$f'(x)$	− 0 +
$f(x)$	↘ 0 ↗

増減表より $f(x) = 0$ は $x = \sqrt{e}$ のみで解をもつ. つまりこの点で曲線①, ②は共有点をもつ.

$y = \dfrac{a}{e}x^2 \to y' = \dfrac{2a}{e}x$

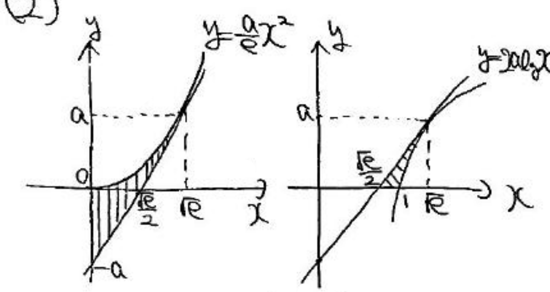
$y = 2a\log x \to y' = \dfrac{2a}{x}$

y'にどちらも$x=\sqrt{e}$を代入すると$\dfrac{2a}{\sqrt{e}}$となるので共通の接線ℓをもつ.

ℓ: $y = \dfrac{2a}{\sqrt{e}}(x - \sqrt{e}) + a$

$\therefore \underline{y = \dfrac{2a}{\sqrt{e}}x - a}\ /\!/$

(2)



$V_1 = \sqrt{e}\sqrt{e}\pi \times 2a \times \dfrac{1}{3} - \displaystyle\int_0^a x^2\pi\, dy$

$= \dfrac{2e\pi a}{3} - \pi\displaystyle\int_0^a \dfrac{e}{a}y\, dy$

$= \dfrac{2e\pi a}{3} - \pi\left[\dfrac{e}{2a}y^2\right]_0^a$

$= \dfrac{2e\pi a}{3} - \pi\dfrac{ea}{2}$

$= \underline{\dfrac{1}{6}e\pi a}\ /\!/$

$V_2 = a^2\pi \times \dfrac{\sqrt{e}}{2} \times \dfrac{1}{3} - \displaystyle\int_1^{\sqrt{e}} (2a\log x)^2\pi\, dx$

$= \dfrac{\sqrt{e}\pi a^2}{6} - 4a^2\pi\displaystyle\int_1^{\sqrt{e}} (\log x)^2 dx$

$= \dfrac{\sqrt{e}\pi a^2}{6}$

$\quad - 4\pi a^2\left[x(\log x)^2 - 2x\log x + 2x\right]_1^{\sqrt{e}}$

$\boxed{(\log x)^2 + 2\log x}\ \boxed{-2\log x - 2}\ \boxed{+2}$

$= \dfrac{\sqrt{e}\pi a^2}{6} - 4\pi a^2\left(\dfrac{\sqrt{e}}{4} - \dfrac{2\sqrt{e}}{2} + 2\sqrt{e} - 2\right)$

$= 8\pi a^2 - \dfrac{29}{6}\sqrt{e}\pi a^2$

$= \underline{\left(8 - \dfrac{29}{6}\sqrt{e}\right)\pi a^2}\ /\!/$

$V_1 = V_2$

$\Leftrightarrow \dfrac{e}{6} = \left(8 - \dfrac{29}{6}\sqrt{e}\right)a$

$\Leftrightarrow e = (48 - 29\sqrt{e})a$

$\Leftrightarrow \underline{a = \dfrac{e}{48 - 29\sqrt{e}}}\ /\!/$