2016 岩手医科大(医)

第1問

問1

$$\left(2^{\frac{\log_2 x}{\log_2 4}}\right)^2-2\log_2 x-8\leqq 0$$

$$\Leftrightarrow (\log_2 x-4)(\log_2 x+2)\leqq 0$$

$$\Leftrightarrow -2\leqq \log_2 x\leqq 4$$

$$\therefore \underline{\frac{1}{4}\leqq x\leqq 16}$$ //

$$(\log_4 x)^2+a\log_4 x-3$$

$$=\frac{1}{4}(\log_2 x)^2+\frac{a}{2}\log_2 x-3$$

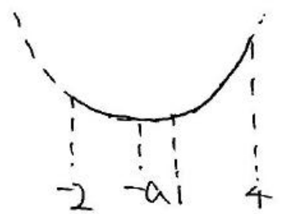
$$=\frac{1}{4}(\log_2 x+a)^2-\frac{a^2}{4}-3$$

$$=\frac{1}{4}(t+a)^2-\frac{a^2}{4}-3 \quad \leftarrow t=\log_2 x$$

$$=f(t) \qquad (-2\leqq t\leqq 4)$$

$f(t)$の$-2\leqq t\leqq 4$での最大値が負になる範囲を求める.

(i) $-a\leqq 1 \Leftrightarrow a\geqq -1$のとき



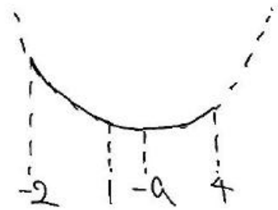
$$\max f(t)=f(4)$$

$$=2a+1<0$$

$$\therefore a<-\frac{1}{2}$$

条件より) $-1\leqq a<-\frac{1}{2}$

(ii) $a\leqq -1$のとき



$$\max f(t)=f(-2)$$

$$=-a-2<0$$

$$\therefore a>-2$$

条件より) $-2<a\leqq -1$

(i)(ii)より) $\underline{-2<a<-\frac{1}{2}}$ //

問2.

$$3x+2y=100$$

$$-)\ 30+2\cdot 50=100$$

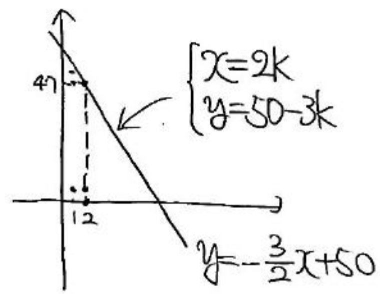
$$3x=2(50-y)$$

$$\downarrow$$

$$\begin{cases} x=2k \\ 50-y=3k \Leftrightarrow y=50-3k \end{cases}$$

$$\therefore 1\leqq k\leqq 16$$

$\underline{16コ}$ //



$$n(3x+2y<100)$$

$$=n(3x+2y\leqq 100)-16$$

$$=n(x\text{が偶数})+n(x\text{が奇数})-16$$

$$=(47+44+\cdots+2)$$

$$+(48+45+\cdots+3)-16$$

$$=(47+44+\cdots+2)\times 2$$

$$=(47+2)\times 16\times\frac{1}{2}\times 2$$

$$=\underline{784コ}$$ //

問3

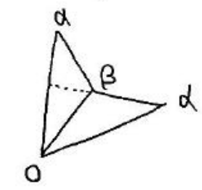
$$\left(\frac{\alpha}{\beta}\right)^2-3\frac{\alpha}{\beta}+3=0$$

$$\Leftrightarrow \frac{\alpha}{\beta}=\frac{3\pm\sqrt{3}i}{2}$$

$$\therefore \underline{\alpha=\beta\left(\frac{3}{2}\pm\frac{\sqrt{3}}{2}i\right)}$$ //

$$=\sqrt{3}\left\{\cos\left(\pm\frac{\pi}{6}\right)+i\sin\left(\pm\frac{\pi}{6}\right)\right\}\beta$$

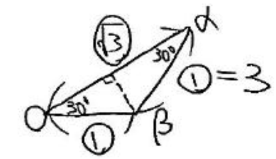
$$|\alpha|=\sqrt{3}|\beta|$$



$$\alpha\bar{\alpha}-\alpha\bar{\beta}-\bar{\alpha}\beta+\beta\bar{\beta}=9$$

$$\Leftrightarrow (\alpha-\beta)(\bar{\alpha}-\bar{\beta})=9$$

$$\Leftrightarrow |\alpha-\beta|^2=9 \quad \therefore |\alpha-\beta|=3$$



$$\triangle OAB=\frac{1}{2}\cdot 3\sqrt{3}\cdot 3\cdot \sin 30^\circ$$

$$=\underline{\frac{9\sqrt{3}}{4}}$$ //

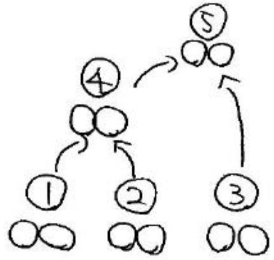
第2問

問1

① ② ③

○○ ○○ ○○

$${}_6C_2 \cdot {}_4C_2 \cdot 1 = \underline{90通り}$$



①〜⑤までの組合せ.

$$90 \times 2^4 = \underline{1440通り}$$

問2

$P(A_1)$

$$= \frac{\overset{①}{⑤⑤}\ \overset{②}{○○}\ \overset{③}{○○}}{90}$$

$$= \frac{1 \cdot {}_4C_2 \cdot 1}{90} = \underline{\frac{1}{15}}$$

$P(A_1 \cap B)$

$$= P(A_1) \cdot \left(\frac{2}{3}\right)^2$$

$$= \underline{\frac{4}{135}}$$

問3

$P(S_1, S_2 \in ① \cap B)$

$$= P(A_1 \cap B) = \frac{4}{135}$$

$P(S_1, S_2 \in ② \cap B)$

$$= P(A_2 \cap B) = \frac{4}{135}$$

$P(S_1, S_2 \in ③ \cap B)$

$$= P(A_3 \cap B) = \frac{1}{15} \cdot \frac{2}{3} = \frac{2}{45}$$

$P(S_1 \in ①, S_2 \in ② \cap B)$

$$= \frac{4 \cdot 3}{90}\left(\underbrace{\frac{2}{3} \cdot \frac{2}{3} \cdot 1 \cdot \frac{2}{3}}_{A_4} + \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot 2\right)$$

$$= \frac{2}{15} \cdot \frac{8}{27} \cdot \frac{5}{3} = \frac{16}{27 \cdot 9}$$

$$P(S_2 \in ①, S_1 \in ① \cap B) = \frac{16}{27 \cdot 9}$$

$P(S_1 \in ①, S_2 \in ③ \cap B)$

$$= \frac{4 \cdot 3}{90}\left(\underbrace{\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}}_{A_5} + \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{3}\right.$$

$$\left. + \left(\frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}\right)\right)$$

$$= \frac{2}{15}\left(\frac{8}{27} + \frac{16}{81} + \frac{4}{27}\right)$$

$$= \frac{2}{15} \cdot \frac{24+16+12}{81} = \frac{104}{15 \cdot 81}$$

$P(S_2 \in ①, S_1 \in ③ \cap B)$

$$= P(S_1 \in ②, S_2 \in ③ \cap B)$$

$$= P(S_2 \in ②, S_1 \in ③ \cap B)$$

$$= \frac{104}{15 \cdot 81}$$

以上より

$$\frac{4}{135} \cdot 2 + \frac{2}{45} + \frac{16}{27 \cdot 9} \cdot 2 + \frac{104}{15 \cdot 81} \cdot 4$$

$$= \frac{72 + 54 + 160 + 416}{1215} = \frac{702}{1215}$$

$$= \underline{\frac{26}{45}}$$

問4

$P(A_4 \cap B)$

$$= P(S_1 \in ①, S_2 \in ② \cap A_4 \cap B) \times 2$$

$$= \frac{4 \cdot 3}{90} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot 2$$

$$= \frac{2}{15} \cdot \frac{8}{27} \cdot 2$$

$\therefore P_B(A_4)$

$$= \frac{P(A_4 \cap B)}{P(B)}$$

$$= \frac{\cancel{2}}{\cancel{15}} \cdot \frac{8}{\cancel{27}_{9}} \cdot 2 \cdot \frac{\cancel{45}^{3}}{\cancel{26}_{13}}$$

$$= \underline{\frac{16}{117}}$$

第3問

問1

$a = \sqrt{3}$ のとき

$$S_1 = \int_0^{\frac{\pi}{2}} (\sqrt{3}-1)\cos x\,dx$$

$$= \left[(\sqrt{3}-1)\sin x\right]_0^{\frac{\pi}{2}}$$

$$= \underline{\sqrt{3}-1}$$

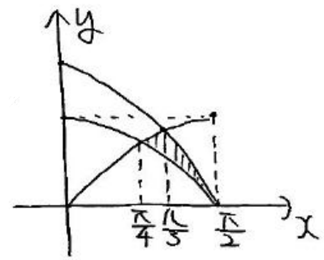
C_1 と C_3 の交点

$$\sin t = \sqrt{3}\cos t$$

$$\Leftrightarrow \tan t = \sqrt{3} \quad \left(0 < t < \frac{\pi}{2}\right)$$

$$\therefore t = \frac{\pi}{3}$$

$$\therefore \cos t = \underline{\frac{1}{2}}$$



S_2

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin x\,dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{3}\cos x\,dx$$

$$- \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\,dx$$

$$= \frac{\sqrt{2}}{2} - \frac{1}{2} + \sqrt{3} - \frac{3}{2} - 1 + \frac{\sqrt{2}}{2}$$

$$= \underline{\sqrt{3} + \sqrt{2} - 3}$$

問2

$$\frac{V_2}{\pi}$$

$$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}(\sin^2 x-\cos^2 x)dx$$

$$+\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}(3\cos^2 x-\cos^2 x)dx$$

$$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}(-\cos 2x)dx$$

$$+\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}(1+\cos 2x)dx$$

$$=\left[-\frac{1}{2}\sin 2x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

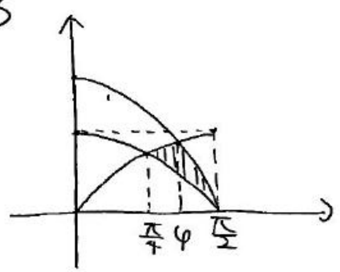
$$+\left[x+\frac{1}{2}\sin 2x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$=-\frac{\sqrt{3}}{4}+\frac{1}{2}+\frac{\pi}{2}-\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right)$$

$$=\frac{\pi}{6}-\frac{\sqrt{3}}{2}+\frac{3}{6}$$

$$\therefore V_2=\frac{\pi(\pi+3-3\sqrt{3})}{6}$$

問3



S_2

$$=\int_{\frac{\pi}{4}}^{\varphi}\sin x\,dx+\int_{\varphi}^{\frac{\pi}{2}}a\cos x\,dx$$

$$-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos x\,dx$$

$$=-\cos\varphi+\frac{\sqrt{2}}{2}+a-a\sin\varphi-1+\frac{\sqrt{2}}{2}$$

$$\sin\varphi=a\cos\varphi$$

$$\tan\varphi=a$$

$$1+\tan^2\varphi=\frac{1}{\cos^2\varphi}$$

$$\therefore \cos\varphi=\frac{1}{\sqrt{a^2+1}}\quad \sin\varphi=\frac{a}{\sqrt{a^2+1}}$$

$$=-\frac{1}{\sqrt{a^2+1}}+a-\frac{a^2}{\sqrt{a^2+1}}-1+\sqrt{2}$$

$$=a-\sqrt{a^2+1}+\sqrt{2}-1$$

$$=\frac{-1}{a+\sqrt{a^2+1}}+\sqrt{2}-1$$

$$\therefore \lim_{a\to\infty}S_2=\sqrt{2}-1$$

問4.

$$S_1=a-1$$

より

$$\lim_{a\to 1+0}\frac{S_2}{S_1}=\lim_{a\to 1+0}\frac{\overbrace{a-\sqrt{a^2+1}}^{f(a)}-\overbrace{(1-\sqrt{2})}^{f(1)}}{a-1}$$

$$=f'(1)$$

$$f'(a)=1-\frac{1}{2}(a^2+1)^{-\frac{1}{2}}2a$$

$$=1-\frac{1}{\sqrt{2}}=\frac{2-\sqrt{2}}{2}$$

問5

$$\frac{V_2}{\pi}$$

$$=\int_{\frac{\pi}{4}}^{\varphi}(\sin^2 x-\cos^2 x)dx$$

$$+\int_{\varphi}^{\frac{\pi}{2}}(a^2\cos^2 x-\cos^2 x)dx$$

$$=\left[-\frac{1}{2}\sin 2x\right]_{\frac{\pi}{4}}^{\varphi}$$

$$+\frac{a^2-1}{2}\int_{\varphi}^{\frac{\pi}{2}}(1+\cos 2x)dx$$

$$=-\frac{1}{2}\sin 2\varphi+\frac{1}{2}$$

$$+\frac{a^2-1}{2}\left\{\frac{\pi}{2}-\left(\varphi+\frac{1}{2}\sin 2\varphi\right)\right\}$$

$$=-\frac{a^2+1}{2}\cdot\frac{1}{2}\sin 2\varphi+\frac{1}{2}+\frac{(a^2-1)\pi}{4}$$

$$-\frac{a^2-1}{2}\varphi$$

$$=\frac{1}{2}\left\{-(a^2+1)\sin\varphi\cos\varphi-(a^2-1)\varphi\right.$$

$$\left.+\frac{\pi}{2}(a^2-1)+1\right\}$$

$$=\frac{1}{2}\left\{(a^2-1)\left(\frac{\pi}{2}-\varphi\right)-(a-1)\right\}$$

$$\frac{V_1}{\pi}$$

$$=(a^2-1)\int_0^{\frac{\pi}{2}}\cos^2 x\,dx$$

$$=(a^2-1)\frac{\pi}{4}$$

$$\lim_{a\to 1+0}\frac{V_2}{V_1}$$

$$=\lim_{a\to 1+0}\frac{2\left\{(a^2-1)\left(\frac{\pi}{2}-\varphi\right)-(a-1)\right\}}{a^2-1}$$

$$=\lim_{a\to 1+0}\frac{2\left\{(a+1)\left(\frac{\pi}{2}-\varphi\right)-1\right\}}{(a+1)\pi}$$

$a\to 1+0$ のとき $\varphi\to\frac{\pi}{4}$ より

$$=\frac{2\left(2\cdot\frac{\pi}{4}-1\right)}{2\pi}$$

$$=\frac{\pi-2}{2\pi}$$