2016 福岡大

[I]

(i)

$$8\,)\,2016$$
$$4\,)\,252$$
$$9\,)\,63$$
$$7$$

$2016 = 2^5 \cdot 3^2 \cdot 7$

(偶数の約数の和)

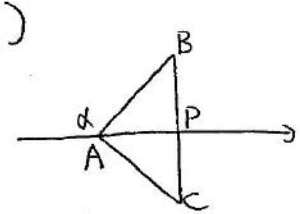
$= (2+2^2+\cdots+2^5)(1+3+3^2)(1+7)$

$= \frac{2-64}{1-2} \cdot 13 \cdot 8$

$= \underline{6448}$ //

(個数) $= 5 \cdot 3 \cdot 2 = \underline{30コ}$ //

(ii)



αは実数解, $\beta = p + qi$, $\gamma = p - qi$

$(q>0)$とする. 解と係数の関係より

$$\begin{cases} \alpha + 2p = 0 \\ \alpha\beta + \beta\gamma + \gamma\alpha = a \\ \alpha(p^2+q^2) = -\frac{5}{2} \end{cases}$$

△ABCが直角二等辺三角形より

$|p - \alpha| = q$.

$\downarrow \alpha = -2p$

$|3p| = q$

$\therefore 9p^2 = q^2$

また

$\alpha(p^2+q^2) = -2p \cdot 10p^2 = -\frac{5}{2}$

$\Leftrightarrow p = \frac{1}{2}$

$\therefore \alpha = -1,\ q = \frac{3}{2}$

$a = \alpha(\beta+\gamma) + p^2 + q^2$

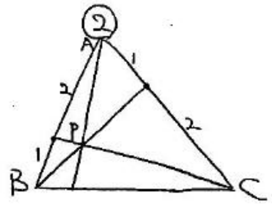
$= -1 + \frac{10}{4} = \underline{\frac{3}{2}}$ //

$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$= 0 - 2a$

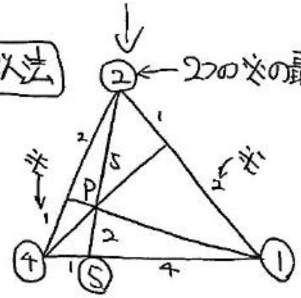
$= \underline{-2a}$ //

(iii)



$\downarrow$

$\boxed{\text{てんびん法}}$



ようするにチェバ・メネラウスの定理

$\overrightarrow{AP} = \frac{5}{7} \cdot \frac{4\overrightarrow{AB} + \overrightarrow{AC}}{1+4}$

$= \frac{4}{7}\overrightarrow{AB} + \frac{1}{7}\overrightarrow{AC}$

$\therefore \underline{s = \frac{4}{7},\ t = \frac{1}{7}}$ //

便宜的に一辺の長さを1とする.

$|\overrightarrow{AP}|^2$

$= \frac{16}{49} + 2\frac{4}{7}\frac{1}{7}\underbrace{\overrightarrow{AB}\cdot\overrightarrow{AC}}_{\frac{1}{2}} + \frac{1}{49}$

$= \frac{21}{49}$

$\therefore |\overrightarrow{AP}| = \frac{\sqrt{21}}{7}$

$\overrightarrow{AB}\cdot\overrightarrow{AP} = |\overrightarrow{AB}||\overrightarrow{AP}|\cos\angle PAB$

$\Leftrightarrow \frac{4}{7} + \frac{1}{14} = \frac{\sqrt{21}}{7}\cos\angle PAB$

$\therefore \cos\angle PAB = \underline{\frac{3\sqrt{21}}{14}}$ //

[II]

(i)

$\left(\frac{1}{2}\right)^{2x} - 12\left(\frac{1}{2}\right)^x + 32 \leqq 0$

$\Leftrightarrow \left\{\left(\frac{1}{2}\right)^x - 4\right\}\left\{\left(\frac{1}{2}\right)^x - 8\right\} \leqq 0$

$\Leftrightarrow 4 \leqq 2^{-x} \leqq 8$

$\Leftrightarrow \underline{-3 \leqq x \leqq -2}$ //

$\log_{10}\left(\frac{1}{24}\right)^{15}$

$= -15(3\log_{10}2 + \log_{10}3)$

$= -20.7015$

$\downarrow$

$-21 < \log_{10}\left(\frac{1}{24}\right)^{15} < -20$

$\Leftrightarrow 10^{-21} < \left(\frac{1}{24}\right)^{15} < 10^{-20}$

小数第21位 //

(ii)

データを$2n$個とする.

○ $\bar{x}$が最大をとるとき

$mA \cdots A | AM \cdots M$

となるので

$\bar{x} = \frac{m + nA + (n-1)M}{2n}$

$= \frac{A+M}{2} - \frac{M-m}{2n}$

$\therefore \bar{x} < \frac{A+M}{2} = 165$

○ $\bar{x}$が最小をとるとき

$m \cdots mA | A \cdots AM$

となるので

$$\bar{x}=\frac{(n-1)m+nA+M}{2n}$$
$$=\frac{m+A}{2}+\frac{M-m}{2n}$$
$$\therefore\ \bar{x}>\frac{m+A}{2}=145$$
以上より $145<\bar{x}<165$ //

また
$$\frac{m+A}{2}<\bar{x}<\frac{A+M}{2}$$
$$\Leftrightarrow m+A<2\bar{x}<A+M$$
$$\Leftrightarrow 2\bar{x}-M<A<2\bar{x}-m$$
$$\Leftrightarrow 160<A<200$$
当然 $A\leqq M$ より
$$160<A\leqq 180$$ //

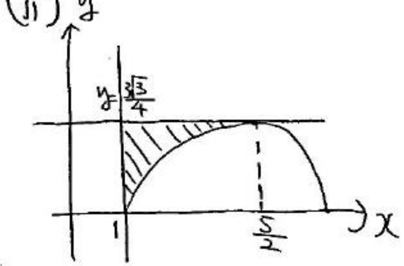
[III] (i)
$$f(x)=(x-1)(-x^2+4x-3)^{\frac{1}{2}}$$
$$f'(x)=\sqrt{-x^2+4x-3}+(x-1)\frac{1}{2}(-x^2+4x-3)^{-\frac{1}{2}}(-2x+4)$$
$$=\frac{-x^2+4x-3+(x-1)(-x+2)}{\sqrt{-x^2+4x-3}}$$
$$=\frac{-2x^2+7x-5}{\sqrt{-x^2+4x-3}}$$
$$=-\frac{(2x-5)(x-1)}{\sqrt{-x^2+4x-3}}$$

x	$1\ \cdots\ \frac{5}{2}\ \cdots\ 3$
$f'(x)$	$\times\ +\ 0\ -$
$f(x)$	$0\ \nearrow\ \frac{3\sqrt{3}}{4}\ \searrow\ 0$

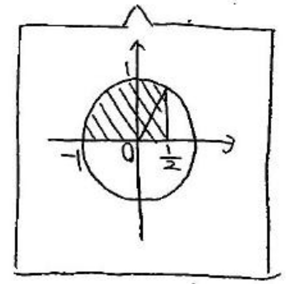
$x=\frac{5}{2}$ で極大値 $\frac{3\sqrt{3}}{4}$

極小値なし //

(ii)



$$=\frac{3}{2}\cdot\frac{3\sqrt{3}}{4}-\int_1^{\frac{5}{2}}(x-1)\sqrt{-x^2+4x-3}\,dx$$
$$=\frac{9\sqrt{3}}{8}-\int_{-1}^{\frac{1}{2}}(t+1)\sqrt{1-t^2}\,dt \quad \swarrow x-2=t$$
$$=\frac{9\sqrt{3}}{8}-\frac{1}{2}\int_{-1}^{\frac{1}{2}}2t\sqrt{1-t^2}\,dt$$
$$-\int_{-1}^{\frac{1}{2}}\sqrt{1-t^2}\,dt$$
$$=\frac{9\sqrt{3}}{8}+\frac{1}{2}\left[(1-t^2)^{\frac{3}{2}}\frac{2}{3}\right]_{-1}^{\frac{1}{2}}$$
$$-1\cdot1\cdot\pi\cdot\frac{1}{3}-\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2}$$



$$=\frac{9\sqrt{3}}{8}+\frac{1}{2}\cdot\frac{3\sqrt{3}}{8}\cdot\frac{2}{3}$$
$$-\frac{\pi}{3}-\frac{\sqrt{3}}{8}$$
$$=\frac{9\sqrt{3}}{8}-\frac{\pi}{3}$$ //