2015 東京医科大

$\boxed{1}$

(1) $|\vec{a}+t\vec{b}|^2=|\vec{c}+t\vec{d}|^2$

$\Leftrightarrow (2+4t)^2+(1+3t)^2=(3+t)^2+(2t)^2$

$\Leftrightarrow 25t^2+22t+5=5t^2+6t+9$

$\Leftrightarrow 20t^2+16t-4=0$

$\Leftrightarrow 5t^2+4t-1=0$

$\Leftrightarrow (5t-1)(t+1)=0$

$\therefore \underline{t_1=-1,\ t_2=\frac{1}{5}}$ //

(2)

$y=x^2$ 上の (t, t^2) での接線は

$y=2t(x-t)+t^2$

$=2tx-t^2$

C_2 と連立

$2tx-t^2=-x^2+18x-53$

$\Leftrightarrow x^2+(2t-18)x+53-t^2=0$

$\frac{D}{4}=(t-9)^2-(53-t^2)$

$=2t^2-18t+28=0$

$\Leftrightarrow t^2-9t+14=0$

$\therefore t=2, 7$

接線 $\underline{y=4x-4,\ y=14x-49}$ //

$\boxed{2}$

(1) $\int_0^1 x^3(1-x^2)^{\frac{3}{2}}dx$

$1-x^2=t$ とおく

$-2xdx=dt$

$\Leftrightarrow xdx=\left(-\frac{1}{2}\right)dt$

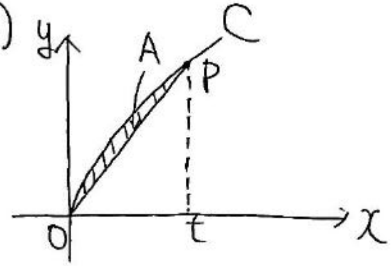
$=\int_1^0 (1-t)t^{\frac{3}{2}}\left(-\frac{1}{2}\right)dt$

$=\frac{1}{2}\int_0^1\left(t^{\frac{3}{2}}-t^{\frac{5}{2}}\right)dt$

$=\frac{1}{2}\left(\frac{2}{5}-\frac{2}{7}\right)$

$=\underline{\frac{2}{35}}$ //

(2)



A

$=\int_0^t\left(\frac{4}{3}x+\frac{2}{3}\sqrt{x}\right)dx$

$-t\times\left(\frac{4}{3}t+\frac{2}{3}\sqrt{t}\right)\times\frac{1}{2}$

$=\left[\frac{2}{3}x^2+\frac{4}{9}x^{\frac{3}{2}}\right]_0^t-\frac{2}{3}t^2-\frac{1}{3}t^{\frac{3}{2}}$

$=\frac{1}{9}t^{\frac{3}{2}}$

S

$=OP\times OP$

$=t^2+\left(\frac{4}{3}t+\frac{2}{3}\sqrt{t}\right)^2$

$=\frac{25}{9}t^2+\frac{16}{9}t^{\frac{3}{2}}+\frac{4}{9}t$

$\frac{A}{S}$

$=\frac{t^{\frac{3}{2}}}{25t^2+16t^{\frac{3}{2}}+4t}$

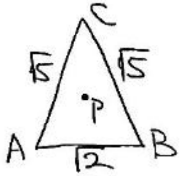
$=\frac{1}{25t^{\frac{1}{2}}+16+4t^{-\frac{1}{2}}}$

$\leqq\frac{1}{2\sqrt{25t^{\frac{1}{2}}\cdot 4t^{-\frac{1}{2}}}+16}$ (相加平均) $\geqq$ (相乗平均)

$=\underline{\frac{1}{36}}$ // (等号成立は $t=\frac{4}{25}$ のとき)

$\boxed{3}$

Pは△ABCの外心を通り、方向ベクトルが平面ABCの法線ベクトルである直線上にあるが、AP^2 が最小値をとるときはPは△ABCの外心であるとき.



余弦定理より

$\cos C=\frac{5+5-2}{2\sqrt{5}\sqrt{5}}=\frac{4}{5}$

$\therefore \sin C=\frac{3}{5}$

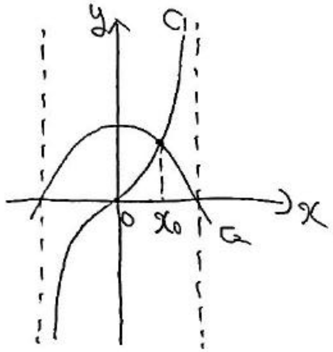
正弦定理より

$2R=\frac{\sqrt{2}}{\sin C}=\frac{5\sqrt{2}}{3}$

$\Leftrightarrow R=\frac{5\sqrt{2}}{6}$

$\therefore m=R^2=\frac{50}{36}=\underline{\frac{25}{18}}$ //

$\boxed{4}$



C_1とC_2連立

$$\tan x = \frac{12}{7}\cos x$$

$$\Leftrightarrow \sin x = \frac{12}{7}\cos^2 x$$

$$\Leftrightarrow \frac{12}{7}\sin^2 x + \sin x - \frac{12}{7} = 0$$

$$\Leftrightarrow 12\sin^2 x + 7\sin x - 12 = 0$$

$$\Leftrightarrow \underbrace{(3\sin x + 4)}_{>0}(4\sin x - 3) = 0$$

$$\therefore \sin x = \underline{\sin x_0 = \frac{3}{4}}$$ //

S

$$= \int_0^{x_0} \left(\frac{12}{7}\cos x - \tan x\right) dx$$

$$= \left[\frac{12}{7}\sin x - \log|\cos x|\right]_0^{x_0}$$

$$= \frac{12}{7}\sin x_0 + \log|\cos x_0|$$

$$= \frac{12}{7}\cdot\frac{3}{4} + \log\frac{\sqrt{7}}{4}$$

$$= \underline{\frac{9}{7} + \frac{1}{2}\log\frac{7}{16}}$$ //