2015 昭和大医 I期

$\boxed{1}$

(1)

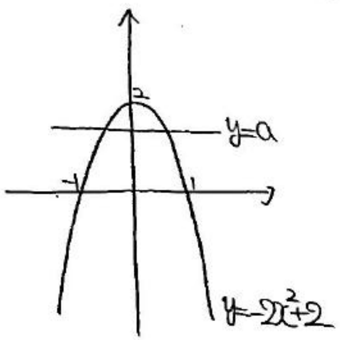
(1-1)

$g(x)-f(x)$

$=x^2+2x+a-(-x^2+2x+2)$

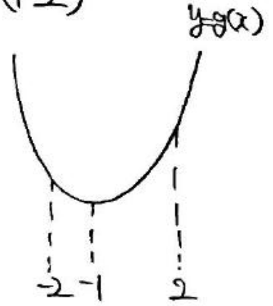
$=2x^2+a-2<0$

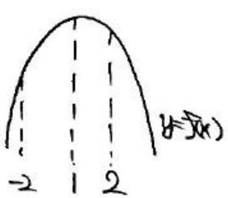
$\Longleftrightarrow\ a<-2x^2+2 \cdots ①$



①をみたす x が $-2\leqq x\leqq 2$ に存在するには $\underline{a<2}$ であればよい.

(1-2)





($g(x)$の最小値)<($f(x)$の最大値)

$\Longleftrightarrow\ a-1<3$

$\therefore\ \underline{a<4}$

(2)

(2-1)

$P_3=\dfrac{{}_4C_2+{}_3C_2}{{}_7C_2}$

$=\dfrac{9}{21}=\underline{\dfrac{3}{7}}$

(2-2)

$P_n=\dfrac{{}_4C_2+{}_nC_2}{{}_{n+4}C_2}$

$=\dfrac{12+n(n-1)}{(n+4)(n+3)}\geqq\dfrac{1}{2}$

$\Longleftrightarrow\ 24+2n^2-2n\geqq n^2+7n+12$

$\Longleftrightarrow\ n^2-9n+12\geqq 0 \quad (n\geqq 2)$

$\min n=\underline{8}$

(3)

$2\sin\left(x+\dfrac{\pi}{3}\right)\geqq\sqrt{2}$

$\Longleftrightarrow\ \sin\left(x+\dfrac{\pi}{3}\right)\geqq\dfrac{1}{\sqrt{2}}$

$\left(\dfrac{\pi}{3}\leqq x+\dfrac{\pi}{3}<\dfrac{7\pi}{3}\right)$

$\dfrac{\pi}{3}\leqq x+\dfrac{\pi}{3}\leqq\dfrac{3}{4}\pi,$

$\dfrac{9}{4}\pi\leqq x+\dfrac{\pi}{3}<\dfrac{7\pi}{3}$

$\Longleftrightarrow\ \underline{0\leqq x\leqq\dfrac{5}{12}\pi,\ \dfrac{23}{12}\pi\leqq x<2\pi}$

(4)

$\log_{10}6^{100}$

$=100\log_{10}6$

$=77.81$

$\therefore\ 77<\log_{10}6^{100}<78$

$\Longleftrightarrow\ 10^{77}<6^{100}<10^{78}$

$\therefore\ \underline{78桁}$

$\boxed{2}$

(1)

群	数列	項数	累計項数
①	(1,1)	1	1
②	(1,2), (2,1)	2	3
③	(1,3), (2,2), (3,1)	3	6
⋮			
ⓚ-1		$k-1$	
ⓚ	(1,k), (2,k-1) (k,1)	k	$\frac{1}{2}k(k+1)$

(5,5)は和が10より9群目の5番目.

$\dfrac{1}{2}\cdot 8\cdot 9+5=\underline{41番目}$

(2)

(m,n)は$m+n-1$群のm番目より

$\dfrac{1}{2}(m+n-2)(m+n-1)+m$

$=\dfrac{1}{2}\{m^2+(2n-3)m+(n-2)(n-1)+2m\}$

$=\underline{\dfrac{1}{2}\{m^2+(2n-1)m+(n-2)(n-1)\}}$

(3)

$\dfrac{1}{2}(k-1)k<200\leqq\dfrac{1}{2}k(k+1)$

$\Longleftrightarrow\ (k-1)k<400\leqq k(k+1)$

$\therefore\ k=20$

200番目は20群の10番目

$\therefore\ \underline{(10,11)}$

(4) k群のC_nの和は

$\displaystyle\sum_{t=1}^{k}t(k+1-t)$

$=\dfrac{1}{2}k(k+1)^2-\dfrac{1}{6}k(k+1)(2k+1)$

$=\dfrac{1}{6}k(k+1)\{3k+3-(2k+1)\}$

$=\dfrac{1}{6}k(k+1)(k+2)$

$C_1+C_2+\cdots+C_{200}$

$=\sum_{k=1}^{19}\frac{1}{6}k(k+1)(k+2)+\sum_{t=1}^{10}t(21-t)$

$=\frac{1}{6}\cdot\frac{1}{4}\cdot19\cdot20\cdot21\cdot22$ 連続自然数積の和

$+21\cdot\frac{1}{2}\cdot10\cdot11-\frac{1}{6}\cdot10\cdot11\cdot21$

$=\underline{8085}$ //

$\boxed{3}$

(1)

(1-1) $\overrightarrow{OA}\cdot\overrightarrow{OB}=\underline{12}$ //

(1-2)

$\cos\angle AOB=\dfrac{\overrightarrow{OA}\cdot\overrightarrow{OB}}{|\overrightarrow{OA}||\overrightarrow{OB}|}$

$=\dfrac{12}{\sqrt{14}\cdot\sqrt{21}}$

$=\underline{\dfrac{2\sqrt6}{7}}$ //

(1-3)

$\triangle OAB$

$=\frac{1}{2}\sqrt{|\overrightarrow{OA}|^2|\overrightarrow{OB}|^2-(\overrightarrow{OA}\cdot\overrightarrow{OB})^2}$

$=\frac{1}{2}\sqrt{14\cdot21-144}=\underline{\frac{5}{2}\sqrt6}$ //

(2) $\frac{1}{x}$の項だけ書くと

${}_9C_2(2x^3)^2\left(-\frac{1}{3x}\right)^7$

$=36\cdot4\left(-\frac{1}{3^7}\right)\frac{1}{x}$

$=\underline{-\frac{16}{243}}\cdot\frac{1}{x}$ //

(3)

$f(x)=\dfrac{(x^2+2)(x^2+3)+5}{x^2+2}$

$=x^2+3+\dfrac{5}{x^2+2}$

$=x^2+2+\dfrac{5}{x^2+2}+1$

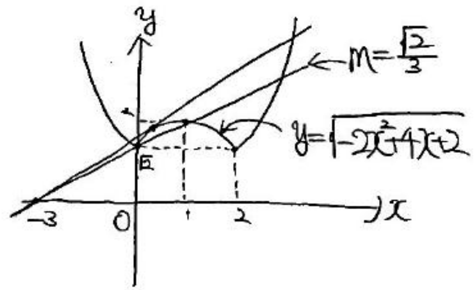
$\geq2\sqrt{(x^2+2)\dfrac{5}{x^2+2}}+1$

$=\underline{2\sqrt5+1}$ //

($x^2=\sqrt5-2$のとき等号成立)

(4)

$y=\sqrt{2+|2x(2-x)|}$



$y=m(x+3)$と$y=\sqrt{-2x^2+4x+2}$

を連立

$m^2(x^2+6x+9)=-2x^2+4x+2$

$\Leftrightarrow(m^2+2)x^2+(6m^2-4)x+9m^2-2=0$

$\frac{D}{4}=(3m^2-2)^2-(m^2+2)(9m^2-2)$

$=-28m^2+8=0$

$\therefore m^2=\frac{2}{7}\quad m=\frac{\sqrt{14}}{7}$

図より求める範囲は

$\underline{\frac{\sqrt2}{3}<m<\frac{\sqrt{14}}{7}}$ //

$\boxed{4}$

(1)

(1-1) $\int_0^1\frac{1}{1+x^2}dx$

$=\left[\tan^{-1}x\right]_0^1=\underline{\frac{\pi}{4}}$ //

(1-2)

S

$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n+3k}{n^2+k^2}$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1+3\frac{k}{n}}{1+\left(\frac{k}{n}\right)^2}$

$=\int_0^1\frac{1+3x}{1+x^2}dx$

$=\frac{\pi}{4}+\frac{3}{2}\int_0^1\frac{2x}{1+x^2}dx$

$=\underline{\frac{\pi}{4}+\frac{3}{2}\log2}$ //

(2)

$\lim_{x\to\pi}\dfrac{\sqrt{a+\cos x}-b}{(x-\pi)^2}$ $x-\pi=t$

$=\lim_{t\to0}\dfrac{\sqrt{a-\cos t}-b}{t^2}$

必要条件 $\sqrt{a-1}-b=0$

$=\lim_{t\to0}\dfrac{a-\cos t-b^2}{t^2(\sqrt{a-\cos t}+b)}$

$=\lim_{t\to0}\dfrac{1-\cos t}{t^2(\sqrt{a-\cos t}+\sqrt{a-1})}\times\dfrac{1+\cos t}{1+\cos t}$

$=\lim_{t\to0}\left(\dfrac{\sin t}{t}\right)^2\dfrac{1}{\sqrt{a-\cos t}+\sqrt{a-1}}\times\dfrac{1}{1+\cos t}$

$=\dfrac{1}{4\sqrt{a-1}}=\dfrac{1}{8}\quad\therefore\sqrt{a-1}=2$

$\therefore\underline{a=5,\ b=2}$ //

これは必要十分条件