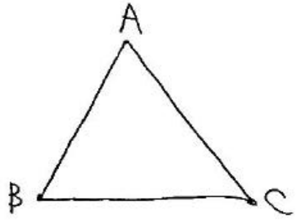
2015 日本医科大

[I]

問1.

(1)



$$\begin{array}{lcl} & n\text{秒後} & n+1\text{秒後} \\ A & P_n \xrightarrow{\frac{1}{4}} & P_{n+1} \\ A\text{以外} & 1-P_n \xrightarrow{\frac{3}{8}} & \end{array}$$

$$P_{n+1} = \frac{1}{4}P_n + \frac{3}{8}(1-P_n)$$
$$= -\frac{1}{8}P_n + \frac{3}{8}$$

(2)

$$\downarrow$$

$$P_{n+1} - \frac{1}{3} = -\frac{1}{8}\left(P_n - \frac{1}{3}\right)$$

↓ 一般項

$$P_n - \frac{1}{3} = \left(P_1 - \frac{1}{3}\right)\left(-\frac{1}{8}\right)^{n-1}$$

$$\therefore P_n = -\frac{1}{12}\left(-\frac{1}{8}\right)^{n-1} + \frac{1}{3}$$

(3)

$$P = \frac{1}{3}$$

(4)

$$\left|P_n - \frac{1}{3}\right|$$
$$= \frac{1}{12}\left(\frac{1}{8}\right)^{n-1} < 5^{-20}$$
$$\downarrow$$
$$\log_{10}\frac{1}{12}\left(\frac{1}{8}\right)^{n-1} < \log_{10}5^{-20}$$
$$\iff -2\log_{10}2 - \log_{10}3 - (3n-3)\log_{10}2 < -20\log_{10}\frac{10}{2}$$
$$\iff -3n\log_{10}2 < -\log_{10}2 + \log_{10}3 - 20 + 20\log_{10}2$$
$$\iff n > -\frac{19}{3} + \frac{20-\log_{10}3}{3\log_{10}2}$$
$$= \frac{19.5229}{0.903} - \frac{19}{3}$$
$$\searrow$$

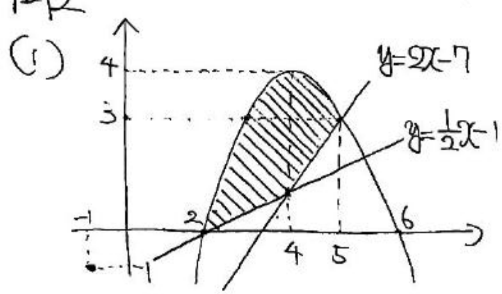
$$\begin{array}{r} 21.62 \\ 903\overline{)19523} \\ 1806 \\ \hline 1463 \\ 903 \\ \hline 5600 \\ 5418 \\ \hline 1820 \end{array}$$

$$\fallingdotseq 21.62 - 6.3$$

$$\therefore \min n = 16$$

問2

(1)



$$\frac{y+1}{(x+1)^2} = k \text{ とおく.}$$

$$\iff y = k(x+1)^2 - 1 \quad \text{頂点}(-1,-1)$$

図より k が最大のとき

$$k(x+1)^2 - 1 = -x^2 + 8x - 12$$
$$\iff (k+1)x^2 + (2k-8)x + k + 11 = 0$$
$$\frac{D}{4} = (k-4)^2 - (k+1)(k+11)$$
$$= -20k + 5 = 0 \quad \therefore k = \frac{1}{4}$$

このとき

$$\frac{5}{4}x^2 - \frac{15}{2}x + \frac{45}{4} = 0$$
$$\iff x^2 - 6x + 9 = 0 \quad \therefore x = 3$$

$\therefore (3, 3)$ のとき $\max k = \frac{1}{4}$

k が最小のときは図からわかりにくいので $y = k(x+1)^2 - 1$ が以下の点を通るときを調べる.

(i) $(2, 0)$ のとき $k = \frac{1}{9} = 0.111\cdots$

(ii) $(4, 1)$ のとき $k = \frac{2}{5^2} = 0.08$

(iii) $(5, 3)$ のとき $k = \frac{4}{6^2} = \frac{1}{9}$

以上より $(4, 1)$ のとき $\min k = \frac{2}{25}$

(2)

$$\frac{y+1}{(x+1)^2} + \frac{(x+1)^2}{y+1}$$
$$= k + \frac{1}{k} = f(k) \quad \left(\frac{2}{25} \leqq k \leqq \frac{1}{4}\right)$$
$$f'(k) = 1 - \frac{1}{k^2} < 0$$

$f(k)$ は $\frac{2}{25} \leqq k \leqq \frac{1}{4}$ で単調減少.

$k = \frac{2}{25}$ のとき 最大値 $\frac{2}{25} + \frac{25}{2}$
$$= \frac{629}{50}$$

$k = \frac{1}{4}$ のとき 最小値 $\frac{1}{4} + 4 = \frac{17}{4}$

[II]

問1

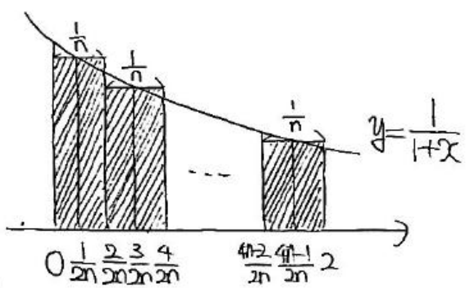
$$\lim_{n\to\infty}\sum_{k=1}^{2n}\frac{1}{n+\frac{k}{2}}$$
$$= \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{2n}\frac{1}{1+\frac{k}{2n}}$$
$$= \lim_{n\to\infty}2\cdot\frac{1}{2n}\sum_{k=1}^{2n}\frac{1}{1+\frac{k}{2n}} \quad \boxed{\text{2n分割で解く}}$$
$$= 2\int_0^1 \frac{1}{1+x}\,dx$$
$$= 2\log 2$$

問2

$$\lim_{n\to\infty}\left(\frac{1}{n+\frac{1}{2}}+\frac{1}{n+\frac{3}{2}}+\cdots+\frac{1}{n+\frac{4n-1}{2}}\right)$$

$$=\lim_{n\to\infty}\sum_{k=1}^{2n}\frac{1}{n+\frac{2k-1}{2}}$$

$$=\lim_{n\to\infty}\sum_{k=1}^{2n}\underbrace{\frac{1}{1+\frac{2k-1}{2n}}}_{\text{ブロックの縦}}\cdot\underbrace{\frac{1}{n}}_{\text{横}}$$



$$=\int_0^2\frac{1}{1+x}dx$$

$$=\underline{\log 3}$$

別

(与式)

$$=\lim_{n\to\infty}\left(\underbrace{\sum_{k=1}^{4n}\frac{1}{n+\frac{k}{2}}}_{\text{全体}}-\underbrace{\sum_{k=1}^{2n}\frac{1}{n+\frac{2k}{2}}}_{\text{偶数番目}}\right)$$

$$=\lim_{n\to\infty}\left(\frac{1}{n}\sum_{k=1}^{4n}\frac{1}{1+\frac{k}{2n}}-\frac{1}{n}\sum_{k=1}^{2n}\frac{1}{1+\frac{k}{n}}\right)$$

$$=\int_0^4\frac{1}{1+\frac{x}{2}}dx-\int_0^2\frac{1}{1+x}dx$$

$$=\cdots=\underline{\log 3}$$

問3

limの中身をAとおく

$$\lim_{n\to\infty}\log A$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\log\left(1+\sin\frac{n+k}{2n}\pi\right)^{\sin\frac{n+k}{n}\pi}$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\sin\frac{n+k}{n}\pi\cdot\log\left(1+\sin\frac{n+k}{2n}\pi\right)$$

$$=\int_1^2\sin\pi x\cdot\log\left(1+\sin\frac{\pi}{2}x\right)dx$$

$$=2\int_1^2\cos\frac{\pi}{2}x\sin\frac{\pi}{2}x\cdot\log\left(1+\sin\frac{\pi}{2}x\right)dx$$

$1+\sin\frac{\pi}{2}x=t$ とおく

$\frac{\pi}{2}\cos\frac{\pi}{2}x\,dx=dt$

$$=\frac{4}{\pi}\int_2^1(t-1)\log t\,dt$$

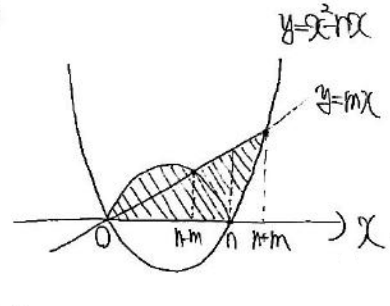
$$=\frac{4}{\pi}\int_2^1(t\log t-\log t)dt$$

$$=\frac{4}{\pi}\left[\frac{t^2}{2}\log t-\frac{t^2}{4}-(t\log t-t)\right]_2^1$$

$$=-\frac{1}{\pi}\qquad\therefore\ \lim A=\underline{e^{-\frac{1}{\pi}}}$$

[II]

問1



V_n

$$=\int_0^{n-m}\pi(x^2-nx)^2dx+\int_{n-m}^{n+m}\pi m^2x^2dx-\int_n^{n+m}\pi(x^2-nx)^2dx$$

$x-n=t$ とおく

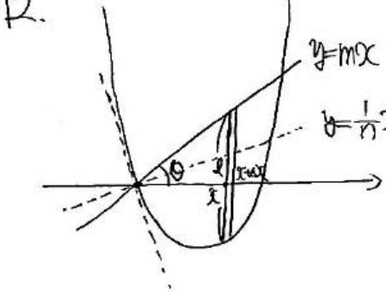
$$=\int_{-n}^{-m}\pi(t+n)^2t^2dt+\int_{-m}^{m}\pi m^2(t+n)^2dt-\int_0^m\pi(t+n)^2t^2dt$$

$$=\pi\left[\frac{t^5}{5}+\frac{2nt^4}{4}+\frac{n^2t^3}{3}\right]_{-n}^{-m}+2\pi m^2\int_0^m(t^2+n^2)dt-\pi\left[\frac{t^5}{5}+\frac{nt^4}{2}+\frac{n^2t^3}{3}\right]_0^m$$

$$=\pi\left(\frac{-2m^5}{5}+\frac{-2n^2m^3}{3}-\left(-\frac{n^5}{5}+\frac{n^5}{2}-\frac{n^5}{3}\right)\right)+2\pi m^2\left(\frac{m^3}{3}+n^2m\right)$$

$$=\underline{\pi\left(\frac{1}{30}n^5+\frac{4}{3}n^2m^3+\frac{4}{15}m^5\right)}$$

問2.



放物線の原点での法線の傾きが$\frac{1}{n}$．$n>\frac{1}{m}\Leftrightarrow m>\frac{1}{n}$より傘型分割で求める．

図のlを母線とした円錐の側面積は

$(2l\cos\theta)\pi$

$$l^2\pi\cdot\frac{(2l\cos\theta)\pi}{2l\pi}=\pi l^2\cos\theta.$$

体積Vのxから$x+\Delta x$までの増分ΔVは，上の円錐の側面積にΔxをかけたものとみなせる．

$\Delta V = \pi l^2 \cos\theta \cdot \Delta x$

これを $x=0$ から $x=n+m$ まで積分すればいいので

W_n

$= \int_0^{n+m} \pi l^2 \cos\theta \, dx$

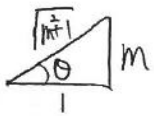
$= \pi \cos\theta \int_0^{n+m} \{(n+m)x - x^2\}^2 dx$

$= \pi \cos\theta$

$\left[\frac{x^5}{5} - \frac{n+m}{2}x^4 + \frac{(n+m)^2}{3}x^3 \right]_0^{n+m}$

$= \pi \cos\theta \cdot \frac{(n+m)^5}{30}$

$= \frac{\pi (n+m)^5}{30\sqrt{m^2+1}}$ //



問3

$\lim_{n\to\infty} \frac{V_n}{W_n}$

$= \lim_{n\to\infty} \frac{\frac{1}{30}n^5 + \frac{4}{3}n^2m^3 + \frac{4}{15}m^5}{\frac{(n+m)^5}{30\sqrt{m^2+1}}}$

$= \sqrt{m^2+1}$ //