2015 日本大(医)

[1]

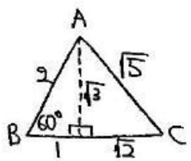
(1)

$$\frac{12}{2+\sqrt{3}+\sqrt{7}}\cdot\frac{2+\sqrt{3}-\sqrt{7}}{2+\sqrt{3}-\sqrt{7}}$$

$$=\frac{12(2+\sqrt{3}-\sqrt{7})}{4\sqrt{3}}$$

$$=\underline{2\sqrt{3}+3-\sqrt{21}}$$ //

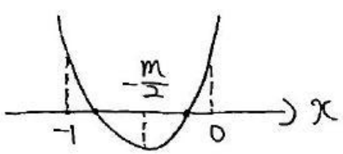
(2)



正 $\frac{\sqrt{15}}{\sin 60^\circ}=2R=\frac{2\sqrt{5}}{\sqrt{3}}=\frac{2\sqrt{15}}{3}$

$\therefore\ R=\underline{\frac{1}{3}\sqrt{15}}$ //

(3)



$$\begin{cases}\text{端}\ f(-1)=-3m+5>0\\ \quad f(0)=-2m+4>0\\ \text{軸}\ -1<-\frac{m}{2}<0\\ \text{判}\ D=m^2-4(-2m+4)>0\end{cases}$$

$$\Leftrightarrow\begin{cases}m<\frac{5}{3}\ ,\ m<2\\ 0<m<2\\ m^2+8m-16>0\end{cases}$$

$\therefore\ \underline{-4+4\sqrt{2}<m<\frac{5}{3}}$ //

(4)

$$\int_0^1(ax^2+x-a^2)dx$$

$$=\frac{a}{3}+\frac{1}{2}-a^2$$

$\underline{a=\frac{1}{6}}$ // のとき最大値 $\frac{1}{18}+\frac{1}{2}-\frac{1}{36}=\underline{\frac{19}{36}}$ //

[2]

(1) 真 $2x+3>0\Leftrightarrow x>-\frac{3}{2}$

$2x+3>\sqrt{5x^2+5x+15}$

$\downarrow$

$4x^2+12x+9>5x^2+5x+15$

$\Leftrightarrow 0>x^2-7x+6$

$\therefore\ \underline{1<x<6}$ //

(2)

$(x-6)^2-36+(y-1)^2-1+33=0.$

$\Leftrightarrow (x-6)^2+(y-1)^2=4$

$y=\frac{1}{6}x+k\Leftrightarrow 0=x-6y+6k$ と

$(6,1)$の距離が2なので

$$\frac{|6-6+6k|}{\sqrt{1+36}}=2$$

$\Leftrightarrow 6|k|=2\sqrt{37}$

$\therefore\ \underline{k=\frac{\sqrt{37}}{3}}$ // $(\because k>0)$

(3) $n\geqq 25$ で $a_n>0$. このときを考える.

$$\frac{a_{n+1}}{a_n}=\frac{n^2+2n+1-23n-23-48}{n^2-23n-48}\cdot\frac{1}{2}$$

$$=\frac{n^2-21n-70}{2n^2-46n-96}>1$$

$\Leftrightarrow 0>n^2-25n-26$

$\Leftrightarrow -1<n<26$

$\downarrow$

$n=25$ のとき $a_{n+1}>a_n$

$n=26$ 〃 $a_{n+1}=a_n$

$n\geqq 27$ 〃 $a_{n+1}<a_n$

$0<a_{25}<a_{26}=a_{27}>\cdots$

a_nが最大となる最大のnは $\underline{27}$ //

(4)

1 2 3 4 5

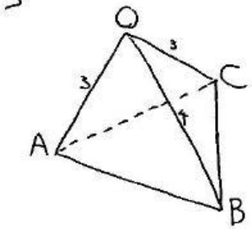
0|| ||0 ||00 ||

$\downarrow$

0×4, |×4の同じものを含む順列より

$$\frac{8!}{4!4!}={}_8C_4=\frac{8\cdot7\cdot6\cdot5}{4\cdot3\cdot2}=\underline{70}\text{とおり}$$ //

[3]



(1)

$\triangle ABC$

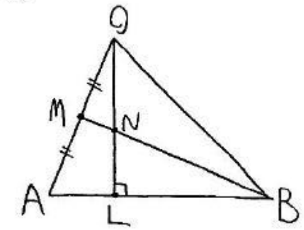
$$=\frac{1}{2}\sqrt{|\vec{BA}|^2|\vec{BC}|^2-(\vec{BA}\cdot\vec{BC})^2}$$

$$=\frac{1}{2}\sqrt{17\cdot17-\{(\vec{a}-\vec{b})\cdot(\vec{c}-\vec{b})\}^2}$$

$$=\frac{1}{2}\sqrt{289-(5-4-4+16)^2}$$

$$=\frac{1}{2}\sqrt{\underbrace{289-169}_{120}}=\underline{\sqrt{30}}$$ //

(2)



$\vec{OL}=t\vec{a}+(1-t)\vec{b}$

$\downarrow$

$\vec{OL}\cdot\vec{BA}=\{t\vec{a}+(1-t)\vec{b}\}(\vec{a}-\vec{b})$

$=9t-4t+4-4t-16(1-t)$

$=17t-12=0$

$\therefore\ t=\frac{12}{17}$

$\therefore \overrightarrow{OL} = \frac{12}{17}\vec{a} + \frac{5}{17}\vec{b}$

$= \frac{24}{17}\cdot\frac{\vec{a}}{2} + \frac{5}{17}\vec{b}$

$= \frac{29}{17}\cdot\underbrace{\frac{24\frac{\vec{a}}{2}+5\vec{b}}{29}}_{\overrightarrow{ON}}$

$\therefore \overrightarrow{ON} = \frac{1}{29}(12\vec{a}+5\vec{b})$ //

(3)

$\overrightarrow{OH} = s\vec{a} + (1-2s)\vec{b} + s\vec{c}$

とおける.

$\overrightarrow{OH}\cdot\overrightarrow{BA}$

$= 9s + 4(1-2s) + 5s$

$\quad -4s - 16(1-2s) - 4s$

$= 30s - 12 = 0 \quad \therefore s = \frac{2}{5}$

$\therefore \overrightarrow{OH} = \frac{1}{5}(2\vec{a}+\vec{b}+2\vec{c})$ //

$|\overrightarrow{OH}|^2$

$= \frac{1}{25}(36+16+36+16+16+40)$

$= \frac{160}{25}$

$\therefore$ (四面体OABC) $= \sqrt{30} \times \frac{\sqrt{160}}{5} \times \frac{1}{3}$

$= \frac{8}{3}\sqrt{3}$ //

[4]

(1) $y' = 1 + \frac{1}{2}(4-(x-1)^2)^{-\frac{1}{2}}(-2(x-1))$

$= 1 - \frac{x-1}{\sqrt{4-(x-1)^2}}$

$y' = 0 \iff \sqrt{4-(x-1)^2} = x-1$

$\downarrow$ 2乗

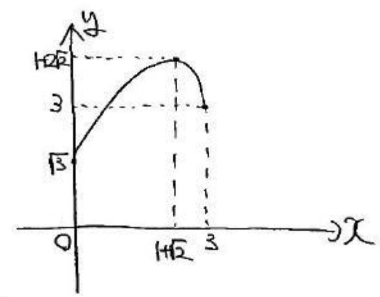
$4-(x-1)^2 = (x-1)^2$

$\iff (x-1)^2 = 2$

$\iff x-1 = \pm\sqrt{2}$

$\therefore x = 1+\sqrt{2} \quad (0 \leqq x \leqq 3)$

x	$0 \cdots 1+\sqrt{2} \cdots 3$
y'	$+ \quad 0 \quad -$
y	$\sqrt{3} \nearrow 1+2\sqrt{2} \searrow 3$



(2)

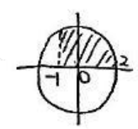
求める面積をSとする.

S

$= \int_0^3 \{x + \sqrt{4-(x-1)^2}\}dx \quad \big) x-1=t$

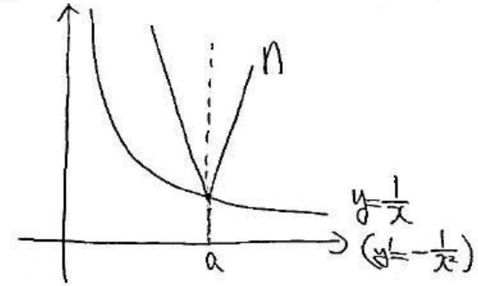
$= \left[\frac{x^2}{2}\right]_0^3 + \int_{-1}^{2}\sqrt{4-t^2}\,dt$

$= \frac{9}{2} + \frac{\sqrt{3}}{2} + 4\pi\cdot\frac{1}{3}$



$= \frac{4}{3}\pi + \frac{9}{2} + \frac{\sqrt{3}}{2}$ //

[5]



$n: y = a^2(x-a) + \frac{1}{a}$

$= a^2x - a^3 + \frac{1}{a}$

$y = f(x)$

$= -a^2(x-a) + \frac{1}{a}$

$= -a^2x + a^3 + \frac{1}{a}$ //

連立

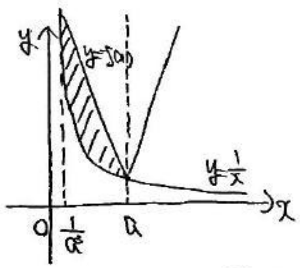
$-a^2x + a^3 + \frac{1}{a} = \frac{1}{x}$

$\iff -a^3x^2 + (a^4+1)x = a$

$\iff 0 = a^3x^2 - (a^4+1)x + a$

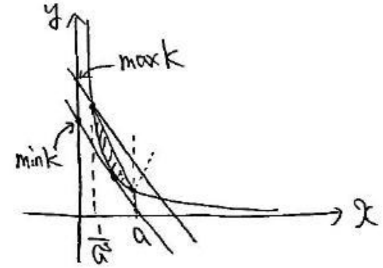
$\iff 0 = (x-a)(a^3x-1)$

$\iff x = a, \frac{1}{a^3}$



領域Dは図の斜線部分.

(2) $x+y=k$とおく. $y=-x+k$.



図より $x=\frac{1}{a^3}$, $y=a^3$のとき

最大値 $\frac{1}{a^3}+a^3$ //

連立

$-x+k = \frac{1}{x}$

$\iff 0 = x^2 - kx + 1$

$D = k^2 - 4 \geqq 0$

$\therefore k \geqq 2 \quad (\because x+y=k>0$

$x=1, y=1$のとき

最小値 2 //