2015 慈恵医大

$\boxed{1}$

(1)

P(Bが勝つ)

$=$ P(4回でBが勝つ)

+P(3回 〃)

+P(2回 〃)

$= {}_3C_2\left(\frac{2}{3}\right)^2\frac{1}{3}\times\frac{1}{3}$

$+ {}_2C_1\frac{2}{3}\cdot\frac{1}{3}\times\frac{1}{3}$

$+\left(\frac{1}{3}\right)^2$

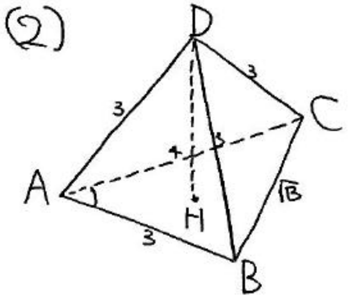
$= \frac{4}{27}+\frac{4}{27}+\frac{3}{27} = \underline{\frac{11}{27}}$

P(3回で終わる)

$=$ P(3回でAが勝つ)+P(3回でBが勝つ)

$= \left(\frac{2}{3}\right)^3 + \frac{4}{27} = \underline{\frac{4}{9}}$

(2)



△DAHと△DBHと△DCHは直角三角形の斜辺とその他の一辺が等しいので合同.

∴ AH=BH=CH=R とおく.

(余)

$\cos A = \frac{3^2+4^2-13}{2\cdot3\cdot4} = \frac{1}{2}$

$\therefore \sin A = \frac{\sqrt{3}}{2}$

(正)

$2R = \frac{\sqrt{13}}{\sin A} = \frac{2\sqrt{13}}{\sqrt{3}}$

$\therefore R = \frac{\sqrt{13}}{\sqrt{3}}$

(三平方)

$DH = \sqrt{3^2 - \frac{13}{3}} = \frac{\sqrt{14}}{\sqrt{3}}$

(体積)

$= \triangle ABC \times DH \times \frac{1}{3}$

$= \frac{1}{2}\cdot4\cdot3\cdot\frac{\sqrt{3}}{2}\times\frac{\sqrt{14}}{\sqrt{3}}\times\frac{1}{3}$

$= \underline{\sqrt{14}}$

$\boxed{2}$

(1) $y = xe^{-x}$

$y' = e^{-x} - xe^{-x}$

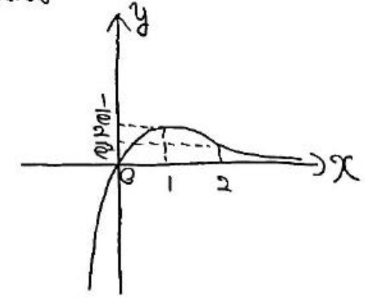
$= (1-x)e^{-x}$

$y'' = -e^{-x} - (1-x)e^{-x}$

$= (x-2)e^{-x}$

x	$\cdots$	1	$\cdots$	2	$\cdots$
y'	+	0	−	−	−
y''	−	−	−	0	+
y	↗		↘		↘

$\lim_{x\to\infty} xe^{-x} = 0$



(2)

(i) $a \geqq 2$ のとき

$S(a)$

$= \int_1^2 (a-x)e^{-x}dx$

ここで

$G(x)$

$= \int (a-x)e^{-x}dx$ とおく

$= -(a-x)e^{-x} + e^{-x} + C$

$\boxed{(a-x)e^{-x}+e^{-x}}\ \boxed{-e^{-x}}$ ← 微分したもの

$= (x-a+1)e^{-x} + C$

より

$S(a)$

$= (3-a)e^{-2} - (2-a)e^{-1}$

$= \underline{\frac{3-a}{e^2} + \frac{a-2}{e}}$

(ii) $1 \leqq a \leqq 2$ のとき

$S(a)$

$= \int_1^a (a-x)e^{-x}dx$

$+ \int_a^2 (x-a)e^{-x}dx$

$= G(a) - G(1) - G(2) + G(a)$

$= 2e^{-a} - (2-a)e^{-1} - (3-a)e^{-2}$

$= \underline{2e^{-a} + \frac{a-2}{e} + \frac{a-3}{e^2}}$

(iii) $0<a\leqq 1$ のとき

$S(a)$

$=\int_1^2 (x-a)e^{-x}dx$

$=\dfrac{a-3}{e^2}-\dfrac{a-2}{e}$ //

(3)

$$S(a)=\begin{cases}\left(\frac{1}{e}-\frac{1}{e^2}\right)a+\frac{3}{e^2}-\frac{2}{e} & \text{(i)}\\ 2e^{-a}+(e^{-1}+e^{-2})a-2e^{-1}-3e^{-2} & \text{(ii)}\\ -\left(\frac{1}{e}-\frac{1}{e^2}\right)a-\frac{3}{e^2}+\frac{2}{e} & \text{(iii)}\end{cases}$$

(ii)のときを調べる.

$S'(a)$

$=-2e^{-a}+e^{-1}+e^{-2}$

$S'(a)=0$

$\Leftrightarrow\ e^{-a}=\dfrac{e^{-1}+e^{-2}}{2}=\dfrac{e+1}{2e^2}$

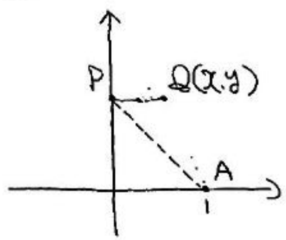
$\Leftrightarrow\ -a=\log_2\dfrac{e+1}{2e^2}$

$\Leftrightarrow\ a=\log_2\dfrac{2e^2}{e+1}$

a	$0\cdots 1\cdots \log_2\frac{2e^2}{e+1}\cdots 2\cdots$
$S'(a)$	$-\ -\ -\ 0\ +\ +\ +$
$S(a)$	$\searrow\ \searrow\ \nearrow\ \nearrow$

$\therefore\ a=\log_2\dfrac{2e^2}{e+1}$ で $S(a)$ は最小 //

$\boxed{3}$



(1) AP+PQ

$=\sqrt{1+y^2}+|x|\leqq n$ //

$D(2):\sqrt{1+y^2}+|x|\leqq 2$

(i) $x\geqq 0$ のとき

$\sqrt{1+y^2}+x\leqq 2$

$\Leftrightarrow\ \underbrace{\sqrt{1+y^2}}_{\geqq 1}\leqq 2-x\quad (x\leqq 1)$

2乗して

$1+y^2\leqq (x-2)^2$

$\Leftrightarrow\ (x-2)^2-y^2\geqq 1$

$(0\leqq x\leqq 1)$

(ii) $x<0$ のとき

$\sqrt{1+y^2}-x\leqq 2$

$\Leftrightarrow\ \underbrace{\sqrt{1+y^2}}_{\geqq 1}\leqq 2+x\quad (x\geqq -1)$

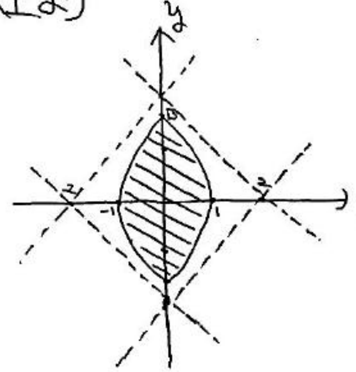
2乗して

$1+y^2\leqq (x+2)^2$

$\Leftrightarrow\ (x+2)^2-y^2\geqq 1$

$(-1\leqq x<0)$

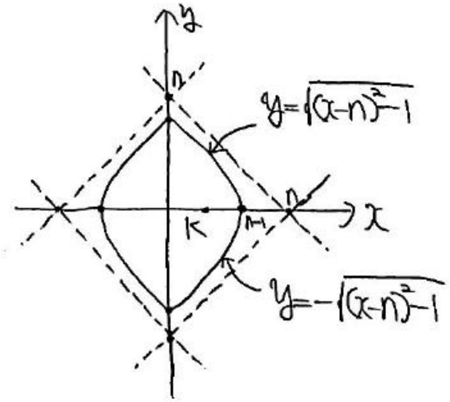
以上より



D(2)は斜線部分.
境界を含む.

(2) (1)より

$$D(n):\begin{cases}(x-n)^2-y^2\geqq 1 & (0\leqq x\leqq n-1)\\ (x+n)^2-y^2\geqq 1 & (-n+1\leqq x<0)\end{cases}$$



$x=k$ のときの格子点を数える.

$y=\sqrt{(x-n)^2-1}<|x-n|$

$x>0$ のとき

$y<n-x$

より $x=k$ のときの格子点の個数は

$n-k-1-(-n+k+1)+1$

$=2n-2k-1.$

y軸対称を考慮して

$S(n)$

$=\underbrace{2n-1}_{x=0\text{のとき}}+2\sum_{k=1}^{n-1}(2n-2k-1)$

$=2n-1$

$+2\{2n(n-1)-(n-1)n-(n-1)\}$

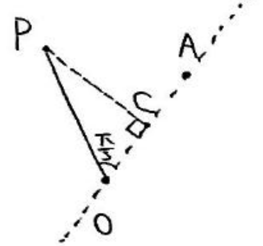
$=2n-1+2(n^2-2n+1)$

$=2n^2-2n+1$ //

$\lim_{n\to\infty}\dfrac{S(n)}{(2n+1)^2}$

$=\lim_{n\to\infty}\dfrac{2-\frac{2}{n}+\frac{1}{n^2}}{\left(2+\frac{1}{n}\right)^2}=\dfrac{1}{2}$ //

$\boxed{4}$



(1)

$C\left(1, \frac{1}{2}, 1\right)$

$\alpha : 2(x-1)+\left(y-\frac{1}{2}\right)+2(z-1)=0$

$\Leftrightarrow 2x+y+2z-\frac{9}{2}=0 \quad \cdots ①$

$\downarrow$ G, H 通る

$$\begin{cases} 2s-\frac{9}{2}=0 \\ 2+2+2t-\frac{9}{2}=0 \end{cases}$$

$\therefore s=\frac{9}{4} \quad t=\frac{1}{4}$

(2) $y-2x=k$ とおいて

$$\begin{cases} x^2+y^2+z^2=9 & (|\vec{OP}|^2=9) \\ z=-x-\frac{1}{2}y+\frac{9}{4} & (①) \\ y=2x+k \end{cases}$$

を連立.

$x^2+(2x+k)^2+\left(-x-x-\frac{k}{2}+\frac{9}{4}\right)^2=9$

$\Leftrightarrow x^2+(2x+k)^2+\left(-2x-\frac{k}{2}+\frac{9}{4}\right)^2=9$

(左辺) − (右辺)

$=5x^2+4kx+k^2$

$\quad +4x^2+\frac{k^2}{4}+\frac{81}{16}+2kx-\frac{9k}{4}-9x-9$

$=9x^2+(6k-9)x+\frac{5}{4}k^2-\frac{9}{4}k-\frac{63}{16}=0$

D

$=(6k-9)^2-36\left(\frac{5}{4}k^2-\frac{9}{4}k-\frac{63}{16}\right)$

$=36k^2-108k+81$

$\quad -45k^2+81k+\frac{9 \cdot 63}{4}$

$=9\left(-k^2-3k+\frac{99}{4}\right) \geqq 0$

$\Leftrightarrow k^2+3k-\frac{99}{4} \leqq 0$

$\therefore \frac{-3-6\sqrt{3}}{2} \leqq y-2x \leqq \frac{-3+6\sqrt{3}}{2}$