2015 北海道大理系(前期)

$\boxed{1}$

(1) $C_1 : y=(x-1)e^x$

$y' = xe^x$

接線

$y = te^t(x-t)+(t-1)e^t$

$= te^t x + (-t^2+t-1)e^t$

これをC_2と連立

$\frac{1}{2e}x^2 + a = te^t x + (-t^2+t-1)e^t$

$\Leftrightarrow$

$\frac{1}{2e}x^2 - te^t x + (t^2-t+1)e^t + a = 0$

$D = (-te^t)^2 - 4\frac{1}{2e}\{(t^2-t+1)e^t + a\}$

$= t^2e^{2t} - 2(t^2-t+1)e^{t-1} - \frac{2a}{e} = 0$

$\therefore \underline{a = \frac{t^2}{2}e^{2t+1} - (t^2-t+1)e^t}$ //

(2) $a = f(t)$とおくと

$f'(t)$

$= te^{2t+1} + t^2e^{2t+1}$

$-(2t-1)e^t - (t^2-t+1)e^t$

$= e^t\{(t^2+t)e^{t+1} - (t^2+t)\}$

$= e^t(e^{t+1}-1)t(t+1)$

t	$\cdots$	-1	$\cdots$	0	$\cdots$
$f'(t)$	$-$	0	$-$	0	$+$
$f(t)$	$\searrow$		$\searrow$	-1	$\nearrow$

$\underline{t=0\text{で極小値}\ -1}$ //

$\boxed{2}$

(1) 与式を両辺p^{n+1}で割ると

$\underbrace{\frac{a_{n+1}}{p^{n+1}}}_{b_{n+1}} = \underbrace{\frac{a_n}{p^n}}_{b_n} + \left(-\frac{q}{p}\right)^{n+1}$

$n \geq 2$のとき

$b_n = b_1 + \sum_{k=1}^{n-1}\left(-\frac{q}{p}\right)^{k+1}$

$= 0 + \frac{\frac{q^2}{p^2} - \left(-\frac{q}{p}\right)^{n+1}}{1+\frac{q}{p}}$

$= \frac{q^2 - p^2\left(-\frac{q}{p}\right)^2\left(-\frac{q}{p}\right)^{n-1}}{p^2+pq}$

$= \frac{q^2}{p(p+q)}\left\{1-\left(-\frac{q}{p}\right)^{n-1}\right\}$

これは$n=1$のときも成立つので

$\underline{b_n = \frac{q^2}{p(p+q)}\left\{1-\left(-\frac{q}{p}\right)^{n-1}\right\}\ (n \geq 1)}$ //

(2)

a_n

$= b_n p^n$

$= \frac{1}{p(p+1)}\left\{1-\left(-\frac{1}{p}\right)^{n-1}\right\}p^n$

$= \frac{1}{p+1}\left\{p^{n-1}-(-1)^{n-1}\right\}$

$(p+1)(a_{n+1}-a_n)$

$= p^n - (-1)^n - p^{n-1} + (-1)^{n-1}$

$= (p-1)p^{n-1} + 2(-1)^{n-1}$

(i) nが奇数のとき

$(p-1)p^{n-1} + 2 \geq 0 \cdots ☆$

① $p \geq 1$のとき ☆は成立.

② $0<p<1$のとき

$0<p^{n-1}<1,\ -1<p-1<0$

より

$-1<(p-1)p^{n-1}<0$

なので ☆は成立.

(ii) nが偶数のとき

$(p-1)p^{n-1} - 2 \geq 0 \cdots ♡$

① $p>1$のとき

$(p-1)p^{n-1}$は$n=2$のとき最小値

をとるので求める条件は

$(p-1)p-2 \geq 0$

$\Leftrightarrow (p+1)(p-2) \geq 0$

$\therefore p \geq 2\ (\because p>0)$

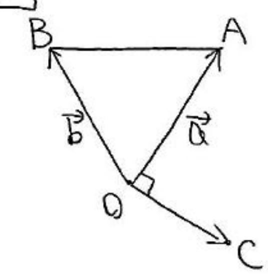
② $0<p<1$のとき

♡は不成立.

以上より(i)(ii)の共通範囲は

$\underline{p \geq 2}$ //

$\boxed{3}$



(1)

$\vec{c} = s\vec{a} + t\vec{b}$

$= \begin{pmatrix} s-t \\ s+t \\ s+t \end{pmatrix}$ とおく

$\vec{a}\cdot\vec{c} = s-t+s+t+s+t$

$= 3s+t = 0 \quad \therefore t=-3s$

$\vec{c} = \begin{pmatrix} 4s \\ -2s \\ -2s \end{pmatrix} \quad |\vec{c}| = \sqrt{16s^2+4s^2+4s^2}$

$= 2\sqrt{6}|s| = 1$

Cのx座標は正なので

$$s=\frac{1}{2\sqrt{6}}$$

$\therefore \underline{C\left(\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)}$ //

(2) (1)より

$$\vec{c}=\frac{1}{2\sqrt{6}}\vec{a}-\frac{3}{2\sqrt{6}}\vec{b}$$

$\vec{b}$について解くと

$$\vec{b}=\frac{1}{3}\vec{a}-\frac{2\sqrt{6}}{3}\vec{c}$$

$\therefore \underline{s=\frac{1}{3} \quad t=-\frac{2\sqrt{6}}{3}}$ //

(3)

$\overrightarrow{PH}=k\vec{a}+l\vec{c}-\overrightarrow{OP}$

$\overrightarrow{PH}\perp\vec{a}$, $\overrightarrow{PH}\perp\vec{b}$ より

$$\begin{cases} k|\vec{a}|^2+l\vec{a}\cdot\vec{c}-\overrightarrow{OP}\cdot\vec{a}=0 \\ k\vec{a}\cdot\vec{b}+l\vec{b}\cdot\vec{c}-\overrightarrow{OP}\cdot\vec{b}=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} 3k-(x+y+z)=0 \\ k-\frac{4}{\sqrt{6}}l-(-x+y+z)=0 \end{cases}$$

$\therefore \underline{k=\frac{x+y+z}{3}}$ //

$$\frac{4x-2y-2z}{3}=\frac{4}{\sqrt{6}}l$$

$\therefore \underline{l=\frac{2x-y-z}{\sqrt{6}}}$ //

$\boxed{4}$

(1)

$P(X_1=3)$

$=P(\text{赤1つ, 白1つ引く})$

$=\frac{2\cdot 2}{{}_4C_2}=\underline{\frac{2}{3}}$ //

(2)

$$\begin{tikzcd}
 & & R4W2 \\
 & R3W2 \arrow[ur, "\frac{6}{10}"] \arrow[dr, "\frac{4}{10}"] & \\
R2W2 \arrow[ur, "\frac{2}{3}"] \arrow[dr, "\frac{1}{3}"'] & & R3W3 \\
 & R2W3 \arrow[ur, "\frac{6}{10}"] \arrow[dr, "\frac{4}{10}"'] & \\
 & & R2W4
\end{tikzcd}$$

図より

$P(X_2=3)=P(R3W3)$

$=\frac{2}{3}\cdot\frac{2}{5}+\frac{1}{3}\cdot\frac{3}{5}$

$=\underline{\frac{7}{15}}$ //

(3) $P_{X_2=3}(X_1=3)$

$=\frac{P(X_1=3\cap X_2=3)}{P(X_2=3)}$

$=\frac{\frac{2}{3}\cdot\frac{2}{5}}{\frac{7}{15}}=\underline{\frac{4}{7}}$ //

$\boxed{5}$

(1)

$\int_0^{\frac{\pi}{2}}\sin^{n+1}\theta\, d\theta$

$=\int_0^{\frac{\pi}{2}}\sin\theta\cdot\sin^n\theta\, d\theta$

$=\left[-\cos\theta\cdot\sin^n\theta\right]_0^{\frac{\pi}{2}}$

$\quad -\int_0^{\frac{\pi}{2}}(-\cos\theta)n\sin^{n-1}\theta\cos\theta\, d\theta$

$=n\int_0^{\frac{\pi}{2}}(1-\sin^2\theta)\sin^{n-1}\theta\, d\theta$

$\Leftrightarrow$

$(n+1)\int_0^{\frac{\pi}{2}}\sin^{n+1}\theta\, d\theta=n\int_0^{\frac{\pi}{2}}\sin^{n-1}\theta\, d\theta$

$\therefore \int_0^{\frac{\pi}{2}}\sin^{n+1}\theta\, d\theta=\frac{n}{n+1}\int_0^{\frac{\pi}{2}}\sin^{n-1}\theta\, d\theta$

(2)

$f(x)$

$=x\int_0^x(a\sin^{n+1}\theta-\sin^{n-1}\theta)\, d\theta$

$\quad -\int_0^x(a\theta\sin^{n+1}\theta-\theta\sin^{n-1}\theta)\, d\theta$

$f'(x)$

$=\int_0^x(a\sin^{n+1}\theta-\sin^{n-1}\theta)\, d\theta$

$\quad +x(a\sin^{n+1}x-\sin^{n-1}x)$

$\quad -(ax\sin^{n+1}x-x\sin^{n-1}x)$

$=\int_0^x(a\sin^{n+1}\theta-\sin^{n-1}\theta)\, d\theta$

$f'\left(\frac{\pi}{2}\right)$

$=a\int_0^{\frac{\pi}{2}}\sin^{n+1}\theta\, d\theta-\int_0^{\frac{\pi}{2}}\sin^{n-1}\theta\, d\theta$

$=\left(\frac{an}{n+1}-1\right)\underbrace{\int_0^{\frac{\pi}{2}}\sin^{n-1}\theta\, d\theta}_{>0}=0 \quad (\because (1))$

$\Leftrightarrow \frac{an}{n+1}-1=0$

$\Leftrightarrow (a-1)n=1 \cdots ①$

$a>\frac{3}{2}$ より

$1=(a-1)n$

$\quad >\left(\frac{3}{2}-1\right)n=\frac{1}{2}n$

$\Leftrightarrow n<2$

$\therefore \underline{n=1}$ // $(\because n\in\mathbb{N})$

①より $\underline{a=2}$ //

(3) $f(x)=\int_0^x\underbrace{(2\sin^2\theta-1)}_{-\cos 2\theta}\, d\theta$

$=\left[-\frac{1}{2}\sin 2\theta\right]_0^x$

$=-\frac{1}{2}\sin 2x$

$\therefore f(x)=\frac{1}{4}\cos 2x-\frac{1}{4}$

$(\because f(0)=0)$

$\therefore \underline{f\left(\frac{\pi}{2}\right)=-\frac{1}{2}}$ //