2015 福岡大(医)

[I]

(i)

$t=\sqrt{3}\sin\theta+\cos\theta$

$=2\sin\left(\theta+\frac{\pi}{6}\right)$

$\downarrow$ $0\leqq\theta\leqq\pi$

$\underline{-1\leqq t\leqq 2}$ //

$K=3\sin^2\theta+2\sqrt{3}\sin\theta\cos\theta+\cos^2\theta$

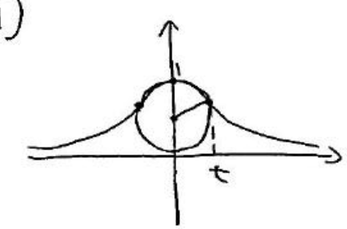
$-1+2t-5$

$=t^2+2t-6$

$=(t+1)^2-7\quad(-1\leqq t\leqq 2)$

$\therefore\ \underline{-7\leqq K\leqq 2}$ //

(ii)



中心$(0,1-r(t))$より

$r(t)=\sqrt{t^2+\left(e^{-t^2}-1+r(t)\right)^2}$

$\downarrow$ 2乗

$r(t)^2=t^2+e^{-2t^2}+1+r(t)^2$
$-2e^{-t^2}-2r(t)+2r(t)e^{-t^2}$

$\Leftrightarrow(2-2e^{-t^2})r(t)$
$=e^{-2t^2}-2e^{-t^2}+t^2+1$

$\therefore\ r(t)=\dfrac{e^{-2t^2}-2e^{-t^2}+t^2+1}{2-2e^{-t^2}}$

$=\dfrac{(1-e^{-t^2})^2+t^2}{2(1-e^{-t^2})}$

$=\underline{\dfrac{1}{2}\left(1-e^{-t^2}+\dfrac{t^2}{1-e^{-t^2}}\right)}$ //

$\lim\limits_{t\to0}r(t)$

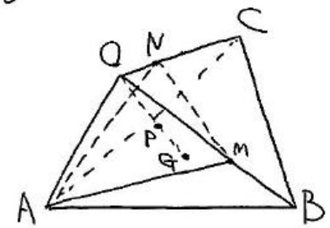
$=\lim\limits_{t\to0}\dfrac{1}{2}\cdot\dfrac{1}{-\dfrac{e^{-t^2}-e^{0}}{t^2-0}}$ $\quad t^2=u$

$=\lim\limits_{u\to0}\dfrac{1}{2}\cdot(-1)\dfrac{1}{\dfrac{e^{-u}-e^{0}}{u-0}}$ $\quad f(u)=e^{-u}$

$=\dfrac{1}{2}(-1)\cdot\underbrace{f'(0)}_{-1}$

$=\dfrac{1}{2}$

(iii)



$\overrightarrow{OA}=\vec{a},\ \overrightarrow{OB}=\vec{b},\ \overrightarrow{OC}=\vec{c}$ とおく.

$\overrightarrow{OG}=\frac{1}{3}(\vec{a}+\vec{b}+\vec{c})$

$\overrightarrow{OP}=(1-s-t)\vec{a}+s\frac{3}{5}\vec{b}+t\frac{1}{5}\vec{c}$

$\overrightarrow{OP}=u\overrightarrow{OG}$ とおくと

$$\begin{cases}1-s-t=\frac{u}{3}\\ \frac{3}{5}s=\frac{u}{3}\\ \frac{1}{5}t=\frac{u}{3}\end{cases}$$

$\downarrow$

$1-\frac{5}{9}u-\frac{5}{3}u=\frac{u}{3}$

$\Leftrightarrow 9-5u-15u=3u$

$\Leftrightarrow\quad u=\frac{9}{23}$

$\therefore\ OP:OG=\underline{9:23}$ //

$\overrightarrow{AP}\cdot\overrightarrow{MN}$

$=(\overrightarrow{OP}-\vec{a})\cdot\left(\frac{1}{5}\vec{c}-\frac{3}{5}\vec{b}\right)$

$=\left(-\frac{20}{23}\vec{a}+\frac{3}{23}\vec{b}+\frac{3}{23}\vec{c}\right)$
$\left(\frac{1}{5}\vec{c}-\frac{3}{5}\vec{b}\right)$

$=\frac{3}{115}|\vec{c}|^2-\frac{9}{115}|\vec{b}|^2=0$

$OB:OC=|\vec{b}|:|\vec{c}|=\underline{1:\sqrt{3}}$ //

[II]

(i)

$2^{-2a+b+1}$

$=2^{\log_2\frac{1}{9}+\log_2 5+\log_2 2}$

$=2^{\log_2\frac{10}{9}}=\frac{10}{9}$

2^{2a-3}

$=2^{\log_2 9+\log_2\frac{1}{8}}=\frac{9}{8}$

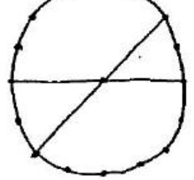
$\therefore\ (2^{-2a+b+1},2^{2a-3})=\underline{\left(\frac{10}{9},\frac{9}{8}\right)}$ //

$2^{-2a+b+1}<2^{-2\times1.584+2.322+1}=2^{0.154}<2^{0.16}$

$2^{2a-3}>2^{2\times1.584-3}=2^{0.168}>2^{0.16}$

$\therefore\ \underbrace{\frac{10}{9}}_{1.111\cdots}<2^{0.16}<\underbrace{\frac{9}{8}}_{1.125}\quad\therefore\ 2^{0.16}\fallingdotseq\underline{1.1}$

(ii)



対角線が直径となれば長方形となる．n本の直径から2本選ぶので

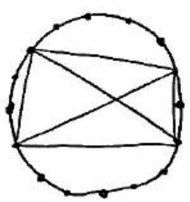
$${}_nC_2 = \frac{1}{2}n(n-1)\text{通り}.$$

n(4つの角がどれも直角でない)

$= n(\text{全体}) - n(\text{2つ直角})$

$\quad - n(\text{4つ直角})$

(i) 2つ直角のとき



n(2つ直角)

$= n$(1本だけ対角線が直径)

$= n(n-1)(n-2)$

n(4つの角がどれも直角でない)

$= {}_{2n}C_4 - n(n-1)(n-2) - \frac{1}{2}n(n-1)$

$= \frac{2n(2n-1)(2n-2)(2n-3)}{4\cdot3\cdot2\cdot1}$

$\quad - n(n-1)\left\{(n-2)+\frac{1}{2}\right\}$

$= \frac{n(n-1)(2n-1)(2n-3)}{6} - \frac{n(n-1)(2n-3)}{2}$

$= \frac{n(n-1)(2n-3)(2n-1-3)}{6}$

$= \frac{1}{3}n(n-1)(n-2)(2n-3)$

III.

$f(x) = \log(1+\sqrt{2+x}) - \frac{1}{2}\sqrt{2+x}$

$f'(x)$

$= \frac{\frac{1}{2}(2+x)^{-\frac{1}{2}}}{1+\sqrt{2+x}} - \frac{1}{4}\cdot\frac{1}{\sqrt{2+x}}$

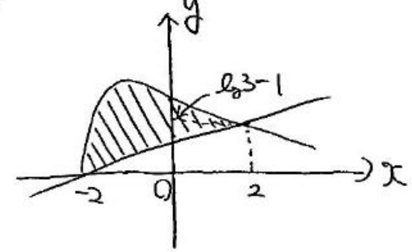
$= \frac{1-\sqrt{2+x}}{4(1+\sqrt{2+x})\sqrt{2+x}}$

x	-2	$\cdots$	-1	$\cdots$
$f'(x)$	$\times$	$+$	0	$-$
$f(x)$	$\times$	$\nearrow$	$\log 2-\frac{1}{2}$	$\searrow$

$x=-1$のとき極大値 $\log 2 - \frac{1}{2}$

極小値なし

(ii)



(面積)

$= \int_{-2}^{2}\left\{\log(1+\sqrt{2+x}) - \frac{1}{2}\sqrt{2+x}\right\}dx$

$\quad - 4\times(\log 3 - 1)\times\frac{1}{2}$

$\sqrt{2+x} = t$
$2+x = t^2$
$dx = 2t\,dt$

$= \int_0^2 \left(2t\log(1+t) - t^2\right)dt$

$\quad - 2(\log 3 - 1)$

$= 2\int_0^2 \left\{(1+t)\log(1+t) - \log(1+t)\right\}dt$

$\quad - \frac{8}{3} - 2\log 3 + 2$

$= 2\left[\frac{1}{2}(1+t)^2\log(1+t) - \frac{1}{4}(1+t)^2 - (1+t)\log(1+t) + t\right]_0^2$

$\quad - 2\log 3 - \frac{2}{3}$

$= 2\left\{\frac{9}{2}\log 3 - \frac{9}{4} - 3\log 3 + 2 + \frac{1}{4}\right\}$

$\quad - 2\log 3 - \frac{2}{3}$

$= \log 3 - \frac{2}{3}$