2014 昭和大医 I期

$\boxed{1}$

(1)

(1-1)

$600 < 3\cdot 2^x < 900$

$\Leftrightarrow 200 < 2^x < 300$

$\therefore \underline{x=8}$

(1-2)

$\frac{7}{2} < \log_2 x < \frac{11}{3}$

$\Leftrightarrow 8\sqrt{2} < x < 8\cdot 2^{\frac{2}{3}}$

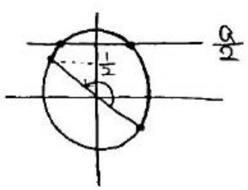
$\therefore \underline{x=12}$

(2)

(2-1)

$2\sin(x-\frac{\pi}{6}) = a$

$\sin(x-\frac{\pi}{6}) = \frac{a}{2}$



図より

$\frac{1}{2} \leqq \frac{a}{2} < 1$

$\therefore \underline{1 \leqq a < 2}$

(2-2)

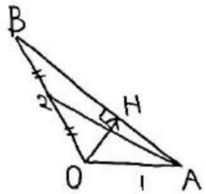
$\tan(\alpha+\beta) = \frac{-\frac{2}{\sqrt{3}}}{1-\frac{-\sqrt{3}}{\sqrt{3}}}$

$= -\frac{1}{\sqrt{3}}$

$\therefore \underline{\alpha+\beta = \frac{5}{6}\pi}$

(3)

(3-1)



$\overrightarrow{OH} = s\vec{a}+(1-s)\vec{b}$

$\overrightarrow{OH}\cdot\overrightarrow{AB}$

$= (s\vec{a}+(1-s)\vec{b})(\vec{b}-\vec{a})$

$= s\underbrace{\vec{a}\cdot\vec{b}}_{-1} - s + 4(1-s) - (1-s)\vec{a}\cdot\vec{b}$

$= -s-s+4-4s+1-s$

$= -7s+5 = 0 \quad \therefore s = \frac{5}{7}$

$\therefore AH:HB = \underline{2:5}$

(3-2)

メネラウス

$\frac{BO}{OM}\cdot\frac{MC}{CA}\cdot\frac{AH}{HB} = 1$

$\Leftrightarrow \frac{2}{1}\cdot\frac{MC}{CA}\cdot\frac{2}{5} = 1$

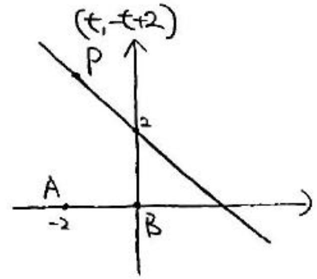
$MC:CA = 5:4$

$\therefore \overrightarrow{OC} = \frac{5\overrightarrow{OA}+4\overrightarrow{OM}}{4+5}$

$= \underline{\frac{5}{9}\vec{a}+\frac{2}{9}\vec{b}}$

$\boxed{2}$

(1)



(1-1) $t=-2$のとき

$\tan\angle APB = \tan\theta = \underline{\frac{1}{2}}$

(1-2)

$\overrightarrow{PA}\cdot\overrightarrow{PB}$

$= \begin{pmatrix}-2-t\\ t-2\end{pmatrix}\cdot\begin{pmatrix}-t\\ t-2\end{pmatrix}$

$= 2t^2-2t+4 = \sqrt{2t^2+8}\sqrt{2t^2-4t+4}\cos\theta$

$\cos\theta = \frac{t^2-t+2}{\sqrt{t^2+4}\sqrt{t^2-2t+2}}$

$1+\tan^2\theta = \frac{(t^2+4)(t^2-2t+2)}{(t^2-t+2)^2}$

$= \frac{t^4-2t^3+6t^2-8t+8}{t^4-2t^3+5t^2-4t+4}$

$\tan^2\theta = \frac{t^2-4t+4}{(t^2-t+2)^2}$

$\therefore \tan\theta = \left|\frac{t-2}{t^2-t+2}\right|$

$= \underline{\frac{|t-2|}{t^2-t+2}}$

(2)

(i) $t<2$のとき

$\tan\theta = \frac{2-t}{t^2-t+2}$

$= -\frac{t-2}{(t-2)(t+1)+4}$

$= -\frac{1}{t+1+\frac{4}{t-2}}$

$= -\frac{1}{t-2+\frac{4}{t-2}+3}$

$= \frac{1}{2-t+\frac{4}{2-t}-3}$

$\leqq \frac{1}{2\sqrt{(2-t)\frac{4}{2-t}}-3} = 1$

$2-t = \frac{4}{2-t} \Leftrightarrow t=0$, $P(0,2)$のとき

$\underline{\max\tan\theta = 1}$

(ii) $t>2$のとき

$$\tan\theta = \frac{t-2}{t^2-t+2}$$
$$= \frac{1}{t-2+\frac{4}{t-2}+3}$$
$$\leqq \frac{1}{2\sqrt{(t-2)\frac{4}{t-2}}+3} = \frac{1}{7}$$

NG!

$\boxed{3}$

(1)

(1-1) $P(R=1)$

$$= P(1,1,2)+P(1,2,2)$$
$$\vdots$$
$$+P(7,7,8)+P(7,8,8)$$
$$= \left(\frac{3}{8^3}+\frac{3}{8^3}\right)\times 7 = \underline{\frac{21}{256}}$$

(1-2)

$P(R=4)$

$$= P(1,\sim,5)+\sim+P(4,\sim,8)$$

$\boxed{\text{〜が1のとき}}$ $\boxed{\text{〜が2〜4のとき}}$ $\boxed{\text{〜が5のとき}}$

$$= \frac{3+3!\times 3+3}{8^3}\times 4$$
$$= \underline{\frac{3}{16}}$$

(1-3)

$P(R=2)$

$$= \frac{3+3!+3}{8^3}\times 6$$

というか一般化します.

$P(R=i)$

$$= \frac{3+6(i-1)+3}{8^3}\times(8-i)$$
$$= \frac{1}{8^3}6i(8-i)$$
$$= \frac{3}{256}(8i-i^2)$$

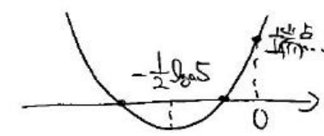
$E(R)$

$$= \sum_{i=1}^{7} iP(R=i)$$
$$= \frac{3}{256}\sum_{i=1}^{7}(8i^2-i^3)$$
$$= \frac{3}{256}\left\{8\cdot\frac{1}{6}7\cdot 8\cdot 15-\frac{1}{4}7^2\cdot 8^2\right\}$$
$$= \frac{3}{4}\left(\frac{35}{2}-\frac{49}{4}\right) = \underline{\frac{63}{16}}$$

(2)

$$f(x) = x^2+(\log_a 5)x+\log_5 a^2$$

とおく.



$$\begin{cases} \text{(端)}\ f(0)=\log_5 a^2>0 \\ \text{(軸)}\ -\frac{1}{2}\log_a 5 = -\frac{1}{2\log_5 a}<0 \\ \text{(判)}\ (\log_a 5)^2-4\log_5 a^2>0 \end{cases}$$

$$\Longleftrightarrow \begin{cases} \text{(端)}\ a^2>1 \cdots ① \\ \text{(軸)}\ a>1 \cdots ② \\ \text{(判)}\ \frac{1}{(\log_5 a)^2}-4\log_5 a^2>0 \end{cases}$$

①,②より $a>1$ なので

(判) $\frac{1}{(\log_5 a)^2}>8\log_5 a$

$\Longleftrightarrow (\log_5 a)^3<\frac{1}{8}$

$\log_5 a$ は正なので

$\log_5 a<\frac{1}{2}$

$\therefore a<\sqrt{5}$

求める条件は

$\underline{1<a<\sqrt{5}}$

(3)(3-1)

$A=B^2=A^4 \cdots ①$

$\det A = a^2+b^2>0$ より

Aは逆行列をもつ. ①にA^{-1}をかけて

$A^3=E=\underline{\begin{pmatrix}1&0\\0&1\end{pmatrix}}$

(3-2)

$$A^2=\begin{pmatrix}a&b\\-b&a\end{pmatrix}\begin{pmatrix}a&b\\-b&a\end{pmatrix}$$
$$=\begin{pmatrix}a^2-b^2&2ab\\-2ab&a^2-b^2\end{pmatrix}$$
$$A^3=\begin{pmatrix}a^3-3ab^2&3a^2b-b^3\\-3a^2b+b^3&a^3-3ab^2\end{pmatrix}=E$$
$$\downarrow$$
$$\begin{cases} a^3-3ab^2=1 \\ 3a^2b-b^3=0 \Longleftrightarrow b^2=3a^2 \end{cases}$$

解くと $\underline{a=-\frac{1}{2},\ b=\frac{\sqrt{3}}{2}}$ (b>

$\boxed{4}$

(1)
(1-1) $a_n=\int_0^1 x^2(1-x)^n dx$

$= \dfrac{2!\,n!}{(n+3)!}$ $\boxed{\text{ベータ関数の公式}}$

$= \underline{\dfrac{2}{(n+1)(n+2)(n+3)}}$ //

(1-2)

$\displaystyle\sum_{k=1}^{n}(k+c)(a_k-a_{k+1})$

$\displaystyle=\sum_{k=1}^{n}\{(k+c)a_k-(k+1+c)a_{k+1}+a_{k+1}\}$

$\displaystyle=(1+c)a_1-(n+1+c)a_{n+1}+\sum_{k=1}^{n}a_{k+1}$

$\boxed{\dfrac{2}{(k+2)(k+3)(k+4)}=\dfrac{1}{(k+2)(k+3)}-\dfrac{1}{(k+3)(k+4)}}$

$=\dfrac{1+c}{12}-(n+1+c)a_{n+1}$

$\quad+\dfrac{1}{12}-\dfrac{1}{(n+3)(n+4)}$

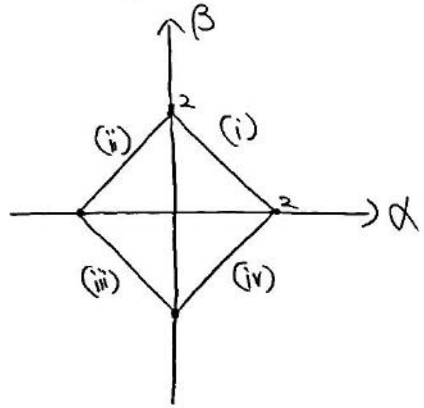
$\displaystyle\therefore \sum_{n=1}^{\infty}(n+c)(a_n-a_{n+1})$

$=\dfrac{2+c}{12}=2 \quad \therefore \underline{c=22}$ //

(2)

$\begin{cases}2x+y=\alpha\\2x-y=\beta\end{cases}$ とおくと

$|\alpha|+|\beta|$ のグラフは



(i) $\alpha+\beta=2$ のとき

$4x=2 \quad \therefore x=\dfrac{1}{2}$

$2y=\alpha-\beta$

$\quad=2\alpha-2 \quad \therefore y=\alpha-1$

$(0\leqq\alpha\leqq 2)$

(ii) $-\alpha+\beta=2$

$4x=\alpha+\beta$

$\quad=2\alpha+2$

$\therefore x=\dfrac{\alpha+1}{2} \quad (-2\leqq\alpha\leqq 0)$

$2y=\alpha-\beta=-2$

$\therefore y=-1$

(iii) $-\alpha-\beta=2$ のとき

$4x=\alpha+\beta=-2$

$\therefore x=-\dfrac{1}{2}$

$2y=\alpha-\beta=2\alpha+2$

$\therefore y=\alpha+1 \quad (-2\leqq\alpha\leqq 0)$

(iv) $\alpha-\beta=2$ のとき

$4x=\alpha+\beta=2\alpha-2$

$\therefore x=\dfrac{\alpha-1}{2} \quad (0\leqq\alpha\leqq 2)$

$2y=\alpha-\beta=2$

$\therefore y=1$

以上より