2014 埼玉医科大(前期)

$\boxed{1}$

問1

$x-1=-\sqrt{2}$ ↓2乗

$x^2-2x+1=2$

$x^2-2x-1=0$

$\therefore x-\dfrac{1}{x}=2$

$x^2+\dfrac{1}{x^2}=\left(x-\dfrac{1}{x}\right)^2+2=6$

$x^4+\dfrac{1}{x^4}=\left(x^2+\dfrac{1}{x^2}\right)^2-2=34$

$x^3-\dfrac{1}{x^3}=\left(x-\dfrac{1}{x}\right)\left(x^2+1+\dfrac{1}{x^2}\right)$

$=2\cdot 7=14$

(与式)

$=34-14+6-2+1=\underline{25}$ //

問2

最初の2次方程式の解をα,βとすると

$\begin{cases}\alpha+\beta=-a \cdots ① \\ \alpha\beta=b \cdots ②\end{cases}$

$\begin{cases}\alpha+3+\beta+3=a+b \cdots ③ \\ (\alpha+3)(\beta+3)=-ab \cdots ④\end{cases}$

の4本満たす.

①,③より

$6=2a+b \iff b=6-2a$

①,②,④より

$b-3a+9=-ab$

以上より

$(6-2a)-3a+9=-a(6-2a)$

$\iff 0=2a^2-a-15$

解くと $a=3,-\dfrac{5}{2}$

$\therefore \underline{(a,b)=(3,0),\left(-\dfrac{5}{2},11\right)}$ //

$x^2+3x=0$　　$x^2-\dfrac{5}{2}x+11=0$

$\therefore \underline{x=0,-3}$ //　　↳ $D<0$

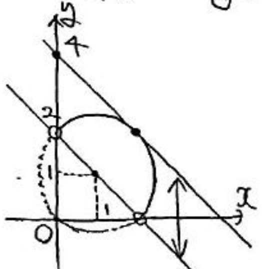
問3.

$(x-1)^2+(y-1)^2=2$

$\log_2(x+y)=M$　$(x>0,\ y>0)$

$\iff x+y=2^M$

$\iff y=-x+2^M$



図より

$2<2^M\leqq 4$

$\therefore \underline{1<M\leqq 2}$ //

問4.

(与式)

$=\lim\limits_{x\to\infty}\dfrac{(8-a^2)x^2+5x-3}{\sqrt{8x^2+5x-3}-ax}$

ここで $8-a^2=0$

$\therefore \underline{a=-2\sqrt{2}}$ //　($\because a>0$だと発散する)

であることが必要.

$=\lim\limits_{x\to\infty}\dfrac{5-\frac{3}{x}}{\sqrt{8+\frac{5}{x}-\frac{3}{x^2}}+2\sqrt{2}}$

$=\dfrac{5}{4\sqrt{2}}=\underline{\dfrac{5\sqrt{2}}{8}}$ //

$\boxed{2}$

問1

$\overrightarrow{AA_1}=(\cos\theta,\sin\theta)$

$\cdots \underline{①,③}$ //

$\overrightarrow{A_1A_2}=\dfrac{1}{2}(\cos(\theta+90^\circ),\sin(\theta+90^\circ))$

$=\dfrac{1}{2}(-\sin\theta,\cos\theta)$

$\cdots \underline{④,①}$ //

$\overrightarrow{A_2A_3}=\dfrac{1}{4}(\cos(\theta+180^\circ),\sin(\theta+180^\circ))$

$=\dfrac{1}{4}(-\cos\theta,-\sin\theta)$

$\cdots \underline{②,④}$ //

$\overrightarrow{A_3A_4}=\dfrac{1}{8}(\cos(\theta+270^\circ),\sin(\theta+270^\circ))$

$=\dfrac{1}{8}(\sin\theta,-\cos\theta)$

$\cdots \underline{③,②}$ //

問2

$\overrightarrow{A_0A_4}=\begin{pmatrix}\frac{3}{4}\cos\theta-\frac{3}{8}\sin\theta \\ \frac{3}{4}\sin\theta+\frac{3}{8}\cos\theta\end{pmatrix}$

$\overrightarrow{A_4A_8}=\dfrac{1}{16}\begin{pmatrix}\frac{3}{4}\cos\theta-\frac{3}{8}\sin\theta \\ \frac{3}{4}\sin\theta+\frac{3}{8}\cos\theta\end{pmatrix}$

↓

$\overrightarrow{A_0A_{4n}}$

$=\left(1+\dfrac{1}{16}+\dfrac{1}{16^2}+\cdots+\dfrac{1}{16^{n-1}}\right)\overrightarrow{A_0A_4}$

$=\dfrac{1-\frac{1}{16^n}}{1-\frac{1}{16}}\overrightarrow{A_0A_4}$

$=\dfrac{16}{15}\left(1-\dfrac{1}{16^n}\right)\dfrac{3}{8}\begin{pmatrix}2\cos\theta-\sin\theta \\ \cos\theta+2\sin\theta\end{pmatrix}$

$=\underline{\dfrac{2}{5}\left(1-\dfrac{1}{4^{2n}}\right)(2\cos\theta-\sin\theta,\ 1\cdot\cos\theta+2\sin\theta)}$ //

3

問1

$$\frac{dy}{d\theta}$$

$$= 2\cos\theta(-\sin\theta)\tan\frac{\theta}{2} + \frac{1}{2}\cos^2\theta\cdot\frac{1}{\cos^2\frac{\theta}{2}}$$

$$= \frac{\cos\theta}{2\cos^2\frac{\theta}{2}}\left(-4\sin\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \cos\theta\right)$$

$$= \frac{\cos\theta}{2\cos^2\frac{\theta}{2}}\left(-2\underbrace{\sin^2\theta}_{1-\cos^2\theta} + \cos\theta\right)$$

$$= \frac{\cos\theta}{2\cos^2\frac{\theta}{2}}\left(2\cos^2\theta + \cos\theta - 2\right)$$

問2

$$\frac{dy}{d\theta} = 0$$

$$\Leftrightarrow 2\cos^2\theta + \cos\theta - 2 = 0$$

$$\Leftrightarrow \cos\theta = \frac{-1+\sqrt{17}}{4}$$

$(\because 0 \leqq \cos\theta \leqq 1)$

θ	0	$\cdots$	α	$\cdots$	$\frac{\pi}{2}$
$\frac{dy}{d\theta}$	$+$	$+$	0	$-$	0
y		$\nearrow$		$\searrow$	

$\left(\cos\alpha = \frac{-1+\sqrt{17}}{4}\right)$

$\cos\theta = \frac{-1+\sqrt{17}}{4}$ のとき最大

問3

$$S$$

$$= \int_0^1 y\,dx$$

$$= \int_{\frac{\pi}{2}}^0 \cos^2\theta\tan\frac{\theta}{2}(-\sin\theta)\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} \cos^2\theta\,\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\,2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} \cos^2\theta(1-\cos\theta)\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} (\cos^2\theta - \cos^3\theta)\,d\theta$$

$$= I(2) - I(3)$$

$\boxed{\text{ウォリス積分}}$

$$I(n) = \frac{n-1}{n}I(n-2)$$

$$= \begin{cases} \frac{(n-1)!!}{n!!}\cdot\frac{\pi}{2} & (n\text{が偶数}) \\ \frac{(n-1)!!}{n!!} & (n\text{が奇数}) \end{cases}$$

$$= \frac{1!!}{2!!}\cdot\frac{\pi}{2} - \frac{2!!}{3!!}$$

$$= \frac{1}{2}\cdot\frac{\pi}{2} - \frac{2}{3\cdot 1}$$

$$= \frac{\pi}{4} - \frac{2}{3}$$

4

問1

$$P(\text{A勝つ})$$

$$= P(\text{A赤, B赤}) + P(\text{A白, B白}) + P(\text{A黄、B黄})$$

$$= \frac{x}{7}\cdot\frac{2}{7} + \frac{y}{7}\cdot\frac{3}{7} + \frac{z}{7}\cdot\frac{2}{7}$$

$$= \frac{2x+3y+2z}{49}$$

問2

$$\frac{2x+3y+2z}{49} \geqq \frac{5}{14}$$

$$\Leftrightarrow 2x+3y+2z \geqq \frac{35}{2}$$

$$\Leftrightarrow 14 + y \geqq \frac{35}{2}$$

$(\because x+y+z=7)$

$$\Leftrightarrow y \geqq \frac{7}{2} \cdots ①$$

$$E$$

$$= 3\cdot\frac{2x}{49} + 1\cdot\frac{3y}{49} + 2\cdot\frac{2z}{49}$$

$$= \frac{1}{49}(6x+3y+4z)$$

$$= \frac{1}{49}(42-3y-2z) \cdots ②$$

①, ②より

$y=4, z=0, x=3$

のとき $\max E = \frac{30}{49}$