2014 日本医科大

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(1)

(i) P(ハート出ない)

$= \frac{{}_nC_2}{{}_9C_2} = \underline{\frac{n(n-1)}{72}}$ //

(ii) P(ハート,スペード1枚ずつ)

$= \frac{(9-n)n}{{}_9C_2} = \underline{\frac{(9-n)n}{36}}$ //

(iii) $\begin{cases} 0.3 \leqq \frac{n(n-1)}{72} < 0.6 \\ \frac{(9-n)n}{36} < 0.4 \end{cases}$

$\Longleftrightarrow \begin{cases} 21.6 \leqq n(n-1) < 43.2 \\ (9-n)n < 14.4 \end{cases}$

これを満たすのは $n = \underline{7}$ //

(2)

(i) $|\overrightarrow{AB}|^2$

$= |\overrightarrow{OB} - \overrightarrow{OA}|^2$

$= \underbrace{|\overrightarrow{OB}|^2}_{3} - \underbrace{2\overrightarrow{OA}\cdot\overrightarrow{OB}}_{2} + \underbrace{|\overrightarrow{OA}|^2}_{3} = 2$

$\therefore AB = \underline{\sqrt{2}}$ //

同様に

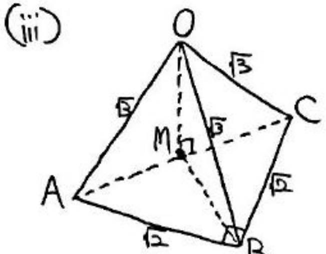
$BC = \underline{\sqrt{2}}$ //

(ii) $\overrightarrow{BA}\cdot\overrightarrow{BC}$

$= (\overrightarrow{OA} - \overrightarrow{OB})\cdot(\overrightarrow{OC} - \overrightarrow{OB})$

$= 1 - 2 - 2 + 3 = 0$

(iii)



$\triangle ABC$が直角二等辺三角形より

$AC = 2$.

↓

ACの中点Mをとると $AM = MC = 1$.

三平方より $OM = \sqrt{2}$.

↓

$MB = 1$なので$\triangle OMB$で三平方の定理の逆より $\angle OMB = 90°$

↓

$(体積) = \sqrt{2} \times \sqrt{2} \times \frac{1}{2} \times \sqrt{2} \times \frac{1}{3}$

$= \frac{\sqrt{2}}{3}$ //

(3)

解と係数

$\begin{cases} \sin\theta + \cos\theta = \frac{2(a+1)}{a^2+1} \\ \sin\theta\cos\theta = \frac{a}{a^2+1} \end{cases}$

$1 = (\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta$

$= \frac{4(a+1)^2}{(a^2+1)^2} - \frac{2a}{a^2+1}$

$\Longleftrightarrow$

$a^4 + 2a^2 + 1$

$= 4a^2 + 8a + 4 - 2a(a^2+1)$

$\Longleftrightarrow$

$a^4 + 2a^3 - 2a^2 - 6a - 3 = 0$

$\Longleftrightarrow (a+1)(a^3 + a^2 - 3a - 3) = 0$

組立除法

1	2	-2	-6	-3	
	-1	-1	3	3	-1
1	1	-3	-3		

$\Longleftrightarrow (a+1)^2(a^2-3) = 0$

$\therefore a = -1, \pm\sqrt{3}$

(i) $a = -1$のとき

$2x^2 - 1 = 0 \quad \therefore x = \pm\frac{1}{\sqrt{2}}$

$\therefore \theta = \frac{3}{4}\pi \quad (\because 0 \leqq \theta \leqq \pi)$

(ii) $a = \sqrt{3}$のとき

$4x^2 - 2(\sqrt{3}+1)x + \sqrt{3} = 0$

$\Longleftrightarrow (2x-1)(2x-\sqrt{3}) = 0$

$\therefore x = \frac{1}{2}, \frac{\sqrt{3}}{2}$

$\therefore \theta = \frac{\pi}{6}, \frac{\pi}{3} \quad (\because 0 \leqq \theta \leqq \pi)$

(iii) $a = -\sqrt{3}$のとき

$4x^2 - 2(-\sqrt{3}+1)x - \sqrt{3} = 0$

$\Longleftrightarrow (2x-1)(2x+\sqrt{3}) = 0$

$\therefore x = \frac{1}{2}, -\frac{\sqrt{3}}{2}$

$\therefore \theta = \frac{5}{6}\pi \quad (\because 0 \leqq \theta \leqq \pi)$

以上より

$(a, \theta) = (-1, \frac{3}{4}\pi), (\sqrt{3}, \frac{\pi}{6}),$

$\underline{(\sqrt{3}, \frac{\pi}{3}), (-\sqrt{3}, \frac{5}{6}\pi)}$ //

2

(1)

$f(x)=\log x-\log k-\frac{1}{k}(x-k)$

とおく.

$f'(x)=\frac{1}{x}-\frac{1}{k}$

x	$0 \cdots k \cdots$
$f'(x)$	× + 0 −
$f(x)$	× ↗ 0 ↘

$\therefore f(x)\leqq 0$

$\therefore \log x-\log k\leqq\frac{1}{k}(x-k)$ //

(2)

(1)の式に $x=a$ を代入

$\log a-\log k\leqq\frac{a}{k}-1$ $\times\alpha$

$\Leftrightarrow \alpha\log a-\alpha\log k\leqq\frac{\alpha a}{k}-\alpha \cdots ①$

同様に

$\beta\log b-\beta\log k\leqq\frac{\beta b}{k}-\beta \cdots ②$

$\gamma\log c-\gamma\log k\leqq\frac{\gamma c}{k}-\gamma \cdots ③$

①, ②, ③ 辺々たすと

$\alpha+\beta+\gamma=1$ を考慮すれば

$\alpha\log a+\beta\log b+\gamma\log c-\log k$

$\leqq\frac{\alpha a+\beta b+\gamma c}{k}-1$

(k = αa+βb+γc とおくと 1)

$\therefore \alpha\log a+\beta\log b+\gamma\log c$

$\leqq\log(\alpha a+\beta b+\gamma c)$ //

(3)

(2)の式に $\alpha=\alpha_1,\ \beta=\beta_1,\ \gamma=\gamma_1$

を代入すると

$\alpha_1\log a+\beta_1\log b+\gamma_1\log c\leqq\log S_1 \cdots ①$

同様に

$\alpha_2\log a+\beta_2\log b+\gamma_2\log c\leqq\log S_2 \cdots ②$

$\alpha_3\log a+\beta_3\log b+\gamma_3\log c\leqq\log S_3 \cdots ③$

①, ②, ③ 辺々たすと

$\log a+\log b+\log c$

$\leqq\log S_1+\log S_2+\log S_3$

$\Leftrightarrow \log abc\leqq\log S_1S_2S_3$

$\Leftrightarrow abc\leqq S_1S_2S_3$ //

3

(1)

(i) $\int_0^2 x^2e^{-x^2}dx$

$=\int_0^2 x\cdot xe^{-x^2}dx$

$=\left[x\left(-\frac{1}{2}e^{-x^2}\right)\right]_0^2-\int_0^2\left(-\frac{1}{2}e^{-x^2}\right)dx$

$=-e^{-4}+\frac{1}{2}I$ //

(ii)

$\int_0^2 x^4e^{-x^2}dx$

$=\int_0^2 x^3\cdot xe^{-x^2}dx$

$=\left[x^3\left(-\frac{1}{2}e^{-x^2}\right)\right]_0^2-\int_0^2 3x^2\left(-\frac{1}{2}e^{-x^2}\right)dx$

$=-4e^{-4}+\frac{3}{2}\left(-e^{-4}+\frac{1}{2}I\right)$

$=-\frac{11}{2}e^{-4}+\frac{3}{4}I$ //

(2) 与式の左辺を b の関数としてみて, $h(b)$ とおく.

$h'(b)$

$=\{f(b)\}^2-\{f'(b)\}^2+\{f''(b)\}^2$

$+2\{f(b)+f'(b)\}\{f'(b)+f''(b)\}$

$=\{f(b)\}^2+\{f'(b)\}^2+\{f''(b)\}^2$

$+2f(b)f'(b)+2f'(b)f''(b)+2f''(b)f(b)$

$=\{f(b)+f'(b)+f''(b)\}^2\geqq 0$

よって $h(b)$ は単調増加.

$h(a)=0$ より $h(b)\geqq 0$

(3)

$f(x)=e^{-\frac{x^2}{2}},\ a=0,\ b=2$ として

(2)を使う.

$f'(x)=-xe^{-\frac{x^2}{2}}$

$f''(x)=(x^2-1)e^{-\frac{x^2}{2}}$

より

$\int_0^2\{1-(-x)^2+(x^2-1)^2\}e^{-x^2}dx$

$+\left[\{1+(-x)\}^2e^{-x^2}\right]_0^2$

$=\int_0^2(2-3x^2+x^4)e^{-x^2}dx$

$+e^{-4}-1$

$=2I-3\left(-e^{-4}+\frac{1}{2}I\right)+\left(-\frac{11}{2}e^{-4}+\frac{3}{4}I\right)$

$+e^{-4}-1$

$=\frac{5}{4}I-\frac{3}{2}e^{-4}-1\geqq 0$

$\therefore \frac{5}{4}I\geqq 1+\frac{3}{2}e^{-4}$

$\therefore I\geqq\frac{4}{5}+\frac{6}{5e^4}$