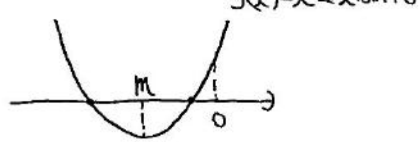
2014 日本大(医)

1.

(1)



$$\begin{cases} \text{端}\ f(0)=5m+6>0 \\ \text{軸}\ m<0 \\ \text{判}\ \dfrac{D}{4}=m^2-(5m+6)>0 \end{cases}$$

$$\Leftrightarrow \begin{cases} m>-\dfrac{6}{5} \\ m<0 \\ m<-1,\ 6<m \end{cases}$$

$\therefore\ \underline{-\dfrac{6}{5}<m<-1}$ //

(2) 平行移動すると

$y+\dfrac{1}{2}=\left(x-\dfrac{1}{2}\right)^2-\left(x-\dfrac{1}{2}\right)+2$

$\Leftrightarrow y=x^2-2x+\dfrac{9}{4}$

↓ 連立

$x=x^2-2x+\dfrac{9}{4}$

$\Leftrightarrow 0=x^2-3x+\dfrac{9}{4}$

$\Leftrightarrow 0=4x^2-12x+9$

$\Leftrightarrow x=\dfrac{3}{2}$

$\therefore\ \underline{\left(\dfrac{3}{2},\dfrac{3}{2}\right)}$ //

(3)

P(赤2コ白2コ)

$=\dfrac{{}_5C_2\cdot{}_4C_2}{{}_9C_4}$

$=\underline{\dfrac{10}{21}}$ //

(4) lとmの交点、(x, y)とおく.

lは

$(x+2)^2+(y-3)^2=(x-1)^2+(y-4)^2$

$\Leftrightarrow 6x+2y=1+16-4-9=4$

mは

$y=-2(x-3)+5=-2x+11$

連立

$3x+(-2x+11)=2$

$\therefore x=-9\quad y=29$

$\therefore\ \underline{(-9, 29)}$ //

(5)

$\dfrac{1}{a_{n+1}}=\dfrac{3a_n+1}{2a_n}=\dfrac{1}{2}\cdot\dfrac{1}{a_n}+\dfrac{3}{2}$

$\Leftrightarrow \dfrac{1}{a_{n+1}}-3=\dfrac{1}{2}\left(\dfrac{1}{a_n}-3\right)$

$\dfrac{1}{a_n}-3=\left(\dfrac{1}{a_1}-3\right)\left(\dfrac{1}{2}\right)^{n-1}$

$\Leftrightarrow \dfrac{1}{a_n}=-2\left(\dfrac{1}{2}\right)^{n-1}+3$

$\Leftrightarrow a_n=\dfrac{1}{3-2\left(\frac{1}{2}\right)^{n-1}}=\underline{\dfrac{2^{n-1}}{3\cdot2^{n-1}-2}}$ //

(6)

$\log_{10}\left(\dfrac{1}{6}\right)^{150}$

$=-150\log_{10}6$

$=-150(0.3010+0.4771)$

$=-116.715$

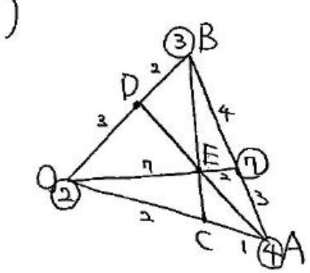
↓

$-117<\log_{10}\left(\dfrac{1}{6}\right)^{150}<-116$

$\Leftrightarrow 10^{-117}<\left(\dfrac{1}{6}\right)^{150}<10^{-116}$

小数第117位 //

(7)



$\overrightarrow{OE}=\dfrac{7}{9}\cdot\dfrac{4\vec{a}+3\vec{b}}{7}$

$=\dfrac{4}{9}\begin{pmatrix}4\\-3\end{pmatrix}+\dfrac{3}{9}\begin{pmatrix}2\\5\end{pmatrix}$

$=\begin{pmatrix}\frac{22}{9}\\ \frac{3}{9}\end{pmatrix}=\underline{\begin{pmatrix}\frac{22}{9}\\ \frac{1}{3}\end{pmatrix}}$ //

(8)

$(x_n)^2+(y_n)^2$

$=(x_n\ y_n)\begin{pmatrix}x_n\\y_n\end{pmatrix}$

$=\left\{A^n\begin{pmatrix}1\\1\end{pmatrix}\right\}'A^n\begin{pmatrix}1\\1\end{pmatrix}$

$=(1\ 1)(A^n)'A^n\begin{pmatrix}1\\1\end{pmatrix}$

$=(1\ 1)(A')^nA^n\begin{pmatrix}1\\1\end{pmatrix}$

$=(1\ 1)(A'A)^n\begin{pmatrix}1\\1\end{pmatrix}$

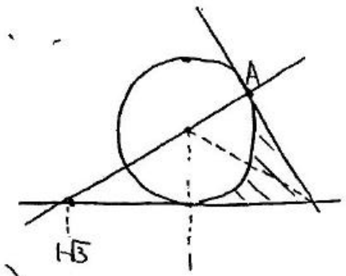
$=(1\ 1)\left\{\begin{pmatrix}1&2\\-2&1\end{pmatrix}\begin{pmatrix}1&-2\\2&1\end{pmatrix}\right\}^n\begin{pmatrix}1\\1\end{pmatrix}$

$=(1\ 1)[5E]^n\begin{pmatrix}1\\1\end{pmatrix}$

$=(1\ 1)\begin{pmatrix}5^n&0\\0&5^n\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}$

$=\underline{2\cdot5^n}$ //

2.

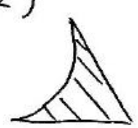


(1)

$l: y = -\sqrt{3}\left(x - \left(1+\frac{\sqrt{3}}{2}\right)\right) + \frac{3}{2}$

$= -\sqrt{3}x + \sqrt{3} + 3$ //

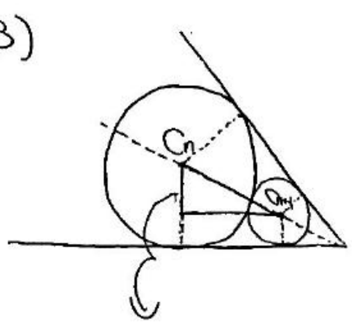
(2)

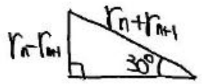


$=$  $\times 2 -$ 

$= \sqrt{3} - \frac{\pi}{3}$ //

(3)





$r_n - r_{n+1} : r_n + r_{n+1} = 1:2$

$\Leftrightarrow 2r_n - 2r_{n+1} = r_n + r_{n+1}$

$\Leftrightarrow r_{n+1} = \frac{1}{3} r_n$

$\therefore r_n = r_1 \left(\frac{1}{3}\right)^{n-1}$

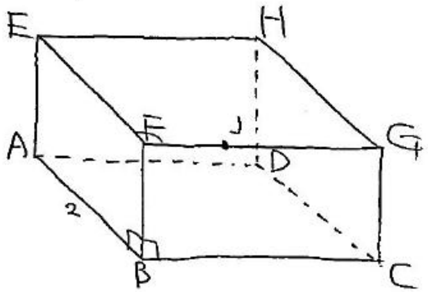
$\therefore S_n = r_1^2 \left(\frac{1}{3}\right)^{2n-2} \pi$

$= \pi r_1^2 \left(\frac{1}{9}\right)^{n-1}$

$\sum_{n=1}^{\infty} S_n = \frac{S_1}{1-\frac{1}{9}}$

$= \frac{\frac{\pi}{9}}{1-\frac{1}{9}} = \frac{\pi}{8}$ //

3.



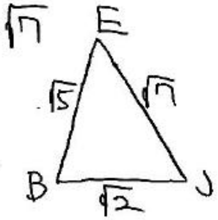
(1)

余

$EJ^2 = 2^2 + 1^2 - 2 \cdot 2 \cdot 1 \cos 120°$

$= 7$

$\therefore EJ = \sqrt{7}$



三平方の定理の逆より

$\angle EBJ = 90°$

$\therefore \triangle EBJ = \sqrt{2} \times \sqrt{5} \times \frac{1}{2} = \frac{\sqrt{10}}{2}$ //

(2)

$\overrightarrow{EK} = s\overrightarrow{EB} + t\overrightarrow{EJ}$ とおく.

また

$\overrightarrow{EK} = \overrightarrow{EF} + \overrightarrow{FK}$

$= \overrightarrow{EF} + k\overrightarrow{FD}$

$\Downarrow$

$s\overrightarrow{EB} + t\overrightarrow{EJ} = \overrightarrow{EF} + k\overrightarrow{FD}$

$\Leftrightarrow s(\overrightarrow{EA} + \overrightarrow{AB}) + t(\overrightarrow{EF} + \overrightarrow{FJ})$

$= \overrightarrow{EF} + k(\overrightarrow{FB} + \overrightarrow{BC} + \overrightarrow{CD})$

$\Leftrightarrow$

$(s-k)\overrightarrow{EA} + (s+t-1+k)\overrightarrow{EF}$

$+ \left(\frac{t}{3} - k\right)\overrightarrow{EH} = \vec{0}$

$\begin{cases} s-k=0 \\ s+t-1+k=0 \\ \frac{t}{3}-k=0 \end{cases}$

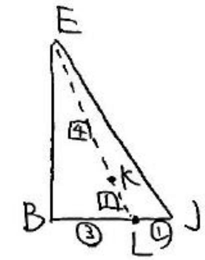
$\Downarrow$

$k+3k-1+k=0 \quad \therefore k = \frac{1}{5}$

$\therefore s = \frac{1}{5} \quad t = \frac{3}{5}$

$\therefore \overrightarrow{EK} = \frac{1}{5}\overrightarrow{EB} + \frac{3}{5}\overrightarrow{EJ}$

(3)



(四面体KFBL)

$= (\text{四面体FBEJ}) \times \frac{3}{4} \times \frac{1}{5}$

$= \frac{1}{2} \cdot 2 \cdot 1 \cdot \sin 120° \cdot 1 \cdot \frac{1}{3} \times \frac{3}{20}$

$= \frac{\sqrt{3}}{40}$

4.

(1)

$$I_1=\int_0^1 \frac{x^2}{1+x^2}dx$$
$$=\int_0^1\left(1-\frac{1}{1+x^2}\right)dx$$
$$=\left[x-\tan^{-1}x\right]_0^1$$
$$=1-\frac{\pi}{4}$$

$$0<\frac{x^{2n}}{1+x^2}<x^{2n}$$
$$\downarrow$$
$$0<I_n<\int_0^1 x^{2n}dx=\frac{1}{2n+1}$$

$\displaystyle\lim_{n\to\infty}\frac{1}{2n+1}=0$ より $\displaystyle\lim_{n\to\infty}I_n=0$

(2)

$$I_n-I_{n+2}$$
$$=\int_0^1\left(\frac{x^{2n}}{1+x^2}-\frac{x^{2n+4}}{1+x^2}\right)dx$$
$$=\int_0^1 x^{2n}\frac{1-x^4}{1+x^2}dx$$
$$=\int_0^1 x^{2n}(1-x^2)dx$$
$$=\int_0^1 (x^{2n}-x^{2n+2})dx$$
$$=\frac{1}{2n+1}-\frac{1}{2n+3}$$
$$=\frac{2}{(2n+1)(2n+3)}$$
$$\downarrow n=2m-1$$
$$I_{2m-1}-I_{2m+1}$$
$$=\frac{2}{(4m-1)(4m+1)}$$

なので

$$\sum_{n=1}^{\infty}\frac{2}{(4n-1)(4n+1)}$$
$$=\sum_{m=1}^{\infty}\left(I_{2m-1}-I_{2m+1}\right)$$
$$=I_1$$
$$=1-\frac{\pi}{4}$$