2014 順天堂医

1

(1) $\begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix}^{-1} = \frac{1}{1}\begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix}$

$= \underline{\begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix}}$

$(1\ -2)A = (-11\ 1)$

$(2\ -2)A = (-8\ 6)$

$\downarrow$

$\begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix}A = \begin{pmatrix} -11 & 1 \\ -4 & 3 \end{pmatrix}$

$\therefore A = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} -11 & 1 \\ -4 & 3 \end{pmatrix}$

$= \underline{\begin{pmatrix} 3 & 5 \\ 7 & 2 \end{pmatrix}}$

(2) $y=\sqrt{x}$ $\quad y'=\frac{1}{2}x^{-\frac{1}{2}}=\underline{\frac{1}{2}}x^{-\frac{1}{2}}$

$(1,1)$での接線は

$y=\frac{1}{2}(x-1)+1$

$=\frac{1}{2}x+\frac{1}{2}$

$\downarrow x=1+h$

$y=\underline{1+\frac{1}{2}h}$

$\sqrt{17}=4\sqrt{\frac{17}{16}}=4\sqrt{\underline{1+\frac{1}{16}}}$

この近似値は $h=\frac{1}{16}$ より

$4\left(1+\frac{1}{2}\cdot\frac{1}{16}\right)=\underline{4}\left(\underline{1+\frac{1}{32}}\right)$

$=\underline{4+\frac{1}{8}}$

ところで

$0<1+\frac{1}{2}h-\sqrt{1+h}<\frac{1}{8}h^2$

$\Leftrightarrow 1+\frac{1}{2}h-\frac{1}{8}h^2<\sqrt{1+h}<1+\frac{1}{2}h$

$\downarrow h=\frac{1}{16}$ 代入し4倍

$4+\frac{1}{8}-\frac{1}{8\cdot 16}<\sqrt{17}<\underbrace{4+\frac{1}{8}}_{4.125}$

これより $\sqrt{17}$ を小数第2位まで求めると

$\underline{4.12}$

$\sqrt{37}=6\sqrt{1+\frac{1}{36}}$

$\fallingdotseq 6\left(1+\frac{1}{2}\cdot\frac{1}{36}\right)$

$=6+\frac{1}{12}$

$=6.0833\cdots$

$$\begin{array}{r} 0.083 \\ 12\,\overline{)100} \\ 96 \\ \hline 40 \\ 36 \\ \hline 4 \end{array}$$

同様に考えて $\sqrt{37}\fallingdotseq\underline{6.08}$

(3)

x軸方向にp, y軸方向にq

移動したものがx軸と$1, 2, \alpha$で

交わるとすると

$(x-p)^3-4(x-p)+q$

$=(x-1)(x-2)(x-\alpha)$

$\Leftrightarrow$

$x^3-3px^2+(3p^2-4)x-p^3+4p+q$

$=x^3-(3+\alpha)x^2+(2+3\alpha)x-2\alpha$

$\downarrow$

$\begin{cases} 3p=3+\alpha \\ 3p^2-4=2+3\alpha \\ -p^3+4p+q=-2\alpha \end{cases}$ 解と係数の関係でもOK

解くと

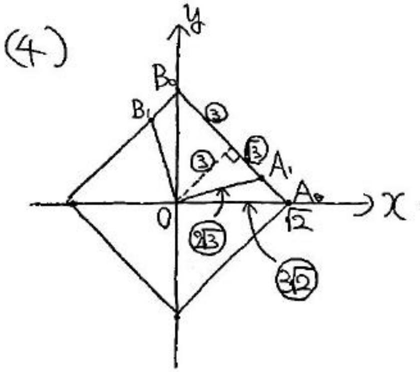
$(p, q, \alpha)=\left(\frac{3\pm\sqrt{5}}{2}, \mp\sqrt{5}, \frac{3\pm3\sqrt{5}}{2}\right)$

より順に

$\frac{3-\sqrt{5}}{2}, \sqrt{5}, \frac{3-3\sqrt{5}}{2}$

$\frac{3+\sqrt{5}}{2}, -\sqrt{5}, \frac{3+3\sqrt{5}}{2}$

(4)



A_1はA_0B_0を$3-\sqrt{3}:3+\sqrt{3}$に内分するので図のようになる.

図より

$\frac{A_1O}{A_0O}=\frac{2\sqrt{3}}{3\sqrt{2}}=\frac{\sqrt{6}}{3}$

一般化しても同じなので

$\frac{A_nO}{A_{n-1}O}=\underline{\frac{\sqrt{6}}{3}}$

正方形$A_1B_1C_1D_1$と$A_0B_0C_0D_0$の相似比は$\sqrt{6}:3$.

$\therefore A_nB_n=A_0B_0\left(\frac{\sqrt{6}}{3}\right)^n$

$=\underline{2\left(\frac{\sqrt{6}}{3}\right)^n}$

$r=\sqrt{2}\left(\frac{\sqrt{6}}{3}\right)^n, \theta=\frac{\pi}{12}n$

より

$r=\sqrt{2}\left(\frac{\sqrt{6}}{3}\right)^{\frac{12}{\pi}\theta}$

$=\sqrt{2}\left(\frac{2}{3}\right)^{\frac{6}{\pi}\theta}$

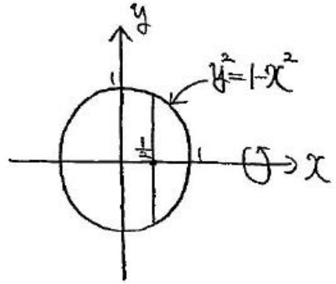
$=\sqrt{2}\,e^{\frac{6}{\pi}\theta\log\frac{2}{3}}$

$\therefore\ a=\sqrt{12}$, $b=\dfrac{6}{\pi}\log\dfrac{2}{3}$

(面積の無限和)

$$=\frac{4}{1-\frac{6}{9}}=12$$

(5)



回転体で直接求める.

$$V_{小}=\int_{\frac{1}{2}}^{1}\pi(1-x^2)dx$$
$$=\pi\left[x-\frac{x^3}{3}\right]_{\frac{1}{2}}^{1}$$
$$=\pi\left\{\frac{2}{3}-\left(\frac{1}{2}-\frac{1}{24}\right)\right\}$$
$$=\frac{5}{24}\pi$$

$V_{全}=\frac{4}{3}\pi$, $V_{大}=\frac{4}{3}\pi-\frac{5}{24}\pi=\frac{9}{8}\pi$

$\therefore$ 小:大 $=5:27$

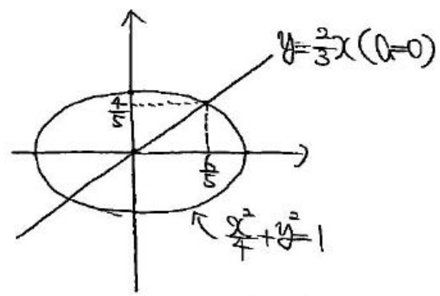
$\frac{1}{4}x^2+y^2=1$ と $y=\frac{2}{3}x+b$ を連立

$$\frac{x^2}{4}+\frac{4x^2}{9}+\frac{4bx}{3}+b^2=1$$
$$\Leftrightarrow 25x^2+48bx+36b^2-36=0$$

$$\frac{D}{4}=576b^2-25(36b^2-36)$$
$$=-324b^2+900=0$$
$$b^2=\frac{900}{324}=\frac{100}{36}=\frac{25}{9}$$
$$\therefore\ y=\frac{2}{3}x-\frac{5}{3}$$

Vは単位球をx軸方向に2倍しただけ

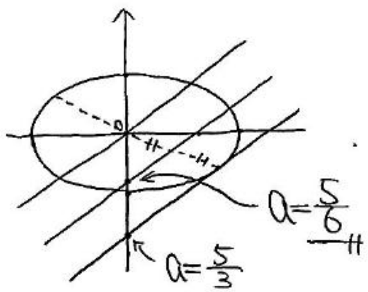
$$V=2V_{全}=\frac{8}{3}\pi$$



断面の楕円の長軸 $2\sqrt{\frac{36}{25}+\frac{16}{25}}$
$$=\frac{4\sqrt{13}}{5}$$

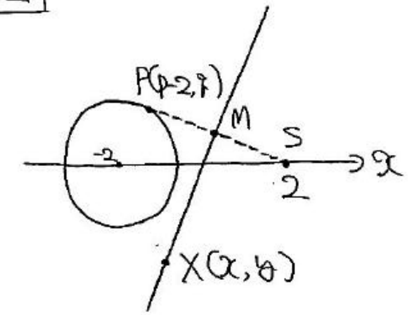
短軸は2より

$$(面積)=\frac{2\sqrt{13}}{5}\cdot 1\cdot\pi=\frac{2\sqrt{13}}{5}\pi$$



ここでカットと 5:27に分ける.

$\boxed{2}$



$$\overrightarrow{SP}\cdot\overrightarrow{MX}$$
$$=\begin{pmatrix}p-4\\q\end{pmatrix}\cdot\begin{pmatrix}x-\frac{p}{2}\\y-\frac{q}{2}\end{pmatrix}$$
$$=(p-4)\left(x-\frac{p}{2}\right)+q\left(y-\frac{q}{2}\right)=0$$
$$\Leftrightarrow qy=-\left(px-\frac{p^2}{2}-4x+2p\right)+\frac{q^2}{2}$$
$$=-p(x+2)+\left(4x+\frac{r^2}{2}\right)$$

↓ 2乗して $q^2=r^2-p^2$

$$p^2\{(x+2)^2+y^2\}-2p(x+2)\left(4x+\frac{r^2}{2}\right)+\left(4x+\frac{r^2}{2}\right)^2-r^2y^2=0$$

$$\frac{D}{4}$$
$$=(x+2)^2\left(4x+\frac{r^2}{2}\right)^2-\{(x+2)^2+y^2\}\left\{\left(4x+\frac{r^2}{2}\right)^2-r^2y^2\right\}$$
$$=(x+2)^2r^2y^2-y^2\left(4x+\frac{r^2}{2}\right)^2+r^2y^4>0$$

↓ $\div y^2$

$$r^2x^2+4r^2x+4r^2-\left(16x^2+4xr^2+\frac{r^4}{4}\right)+r^2y^2>0$$
$$\therefore\ r^2y^2+(r^2-16)x^2-\frac{r^2}{4}(r^2-16)>0$$

↓

$$\frac{x^2}{\frac{r^2}{4}}+\frac{y^2}{\frac{r^2-16}{4}}=1$$

$r>4$ のとき楕円…a

$r<4$ のとき双曲線…c

焦点はいずれも $(-2,0)$, $(2,0)$

$\boxed{3}$

(1) 各内角の二等分線の交点.

(2)

$\vec{u}\cdot\vec{w}$

$=\vec{u}\cdot\vec{v}-(\vec{v}\cdot\vec{u})\underbrace{|\vec{u}|^2}_{1}$

$=0$

$\therefore\ \vec{u}\perp\vec{w}$

(3)

$\vec{x}$

$=k\vec{u}+l\vec{u}-(k\vec{u}\cdot\vec{u}+l)\vec{u}$

$=k\vec{u}-k(\vec{u}\cdot\vec{v})\vec{u}$

$\vec{y}$ 上と同様に

$=l\vec{u}-l(\vec{u}\cdot\vec{v})\vec{v}$

$|\vec{x}|^2=|\vec{y}|^2$

$k^2-2k^2(\vec{u}\cdot\vec{v})^2+k^2(\vec{u}\cdot\vec{v})^2$

$=l^2-2l^2(\vec{u}\cdot\vec{v})^2+l^2(\vec{u}\cdot\vec{v})^2$

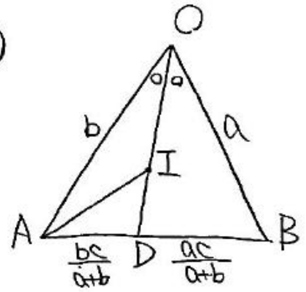
$\Longleftrightarrow k^2\{1-(\vec{u}\cdot\vec{v})^2\}=l^2\{1-(\vec{u}\cdot\vec{v})^2\}$

$\vec{u}$と$\vec{v}$は平行でないので$\vec{u}\cdot\vec{v}\neq 1$

$\therefore\ k^2=l^2$

$\therefore\ \underline{k=l}$ $(\because k,\ l>0)$

(4)



$OI:ID=b:\frac{bc}{a+b}$

$=a+b:c$

$\vec{OI}=\frac{a+b}{a+b+c}\cdot\frac{a\vec{f}+b\vec{g}}{b+a}$

$=\underline{\frac{a\vec{f}+b\vec{g}}{a+b+c}}$