2012 埼玉医科大

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問1 $y+16z=kx \cdots ①$

$2z-x=ky \cdots ②$

$x+2y=kz \cdots ③$

②+③

$2z+2y=ky+kz$

$\Leftrightarrow (2-k)(y+z)=0$

$\therefore k=2,\ z=-y$

$z=-y$のとき

$\begin{cases} -15y=kx \\ -x-2y=ky \Leftrightarrow (-k-2)y=x \end{cases}$

↓

$-15y=(-k^2-2k)y$

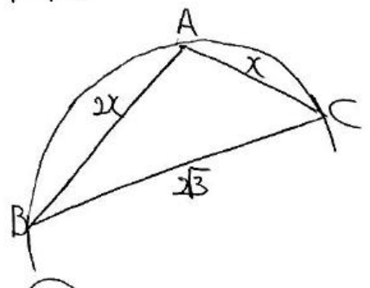
$\therefore -15=-k^2-2k$

$\therefore k^2+2k-15=0$

$\therefore k=3,-5$

以上より $\underline{k=-5,2,3}$ //

問2



正

$\frac{2\sqrt3}{\sin A}=2\cdot2$

$\Leftrightarrow \sin A=\frac{\sqrt3}{2}$

$\therefore A=120°$ (∵AB直径)

余

$12=4x^2+x^2-2\cdot2x\cdot x\underbrace{\cos120°}_{-\frac12}$

$=7x^2$

$x^2=\frac{12}{7}$ $\therefore x=\frac{2\sqrt3}{\sqrt7}$

(面積)

$=\frac{1}{2}\cdot\frac{4\sqrt3}{\sqrt7}\cdot\frac{2\sqrt3}{\sqrt7}\cdot\sin120°$

$=\frac{12}{7}\cdot\frac{\sqrt3}{2}$

$=\frac{6\sqrt3}{7}$

問3.

$3\frac{1-\cos2\theta}{2}-\frac{\sqrt3-1}{2}\sin2\theta+(2\sqrt3)\frac{1+\cos2\theta}{2}=2$

$3-3\cos2\theta-(\sqrt3-1)\sin2\theta+2\sqrt3+(2\sqrt3)\cos2\theta=4$

$(-1+\sqrt3)\cos2\theta+(1-\sqrt3)\sin2\theta=-1+\sqrt3$

$(\sqrt3-1)\sin2\theta+(\sqrt3+1)\cos2\theta=1-\sqrt3$

$\frac{\sqrt3-1}{2\sqrt2}\sin2\theta+\frac{\sqrt3+1}{2\sqrt2}\cos2\theta=\frac{1-\sqrt3}{2\sqrt2}$

$\sin2\theta\times\frac{\sqrt6-\sqrt2}{4}+\cos2\theta\times\frac{\sqrt6+\sqrt2}{4}=\frac{\sqrt2-\sqrt6}{4}$

$\sin\left(2\theta+\frac{5}{12}\pi\right)=-\frac{\sqrt6-\sqrt2}{4}$

$2\theta+\frac{5}{12}\pi=\frac{13}{12}\pi,\ \frac{23}{12}\pi$

$2\theta=\frac{8}{12}\pi,\ \frac{18}{12}\pi$

$\theta=\frac{\pi}{3},\ \frac{3}{4}\pi$

問4.

$\underbrace{(3x+4y-1)}_{0}k\underbrace{-x+2y+7}_{0}=0$

$3x+4y-1=0$

$-3x+6y+21=0$

$10y+20=0$ $\quad y=-2$

$\underline{x=3}$ //

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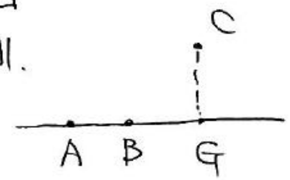
問1.



Aを通り、$\vec{AB}=\begin{pmatrix}1\\1\\1\end{pmatrix}$に平行な直線$l$は

$l: \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}t\\-1+t\\t\end{pmatrix}$

$G(t,-1+t,t)$とおく

$\vec{AB}\cdot\vec{CG}=\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}t\\-4+t\\t-2\end{pmatrix}$

$=3t-6=0$ $\therefore t=2$

$\therefore G(2,1,2)$

問2.

$z=ax+by+c$

↓ A, B, Cを通る

$0=-b+c$

$1=a+c$

$2=3b+c$

$b=\frac12,\ c=\frac12\ \ a=\frac12$

$z=\frac{x}{2}+\frac{y}{2}+\frac{1}{2}$

$\Leftrightarrow x+y-2z+1=0$

$\vec{n}=\begin{pmatrix}1\\1\\-2\end{pmatrix}$に平行でOを通る直線は

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} k \\ k \\ -2k \end{pmatrix}$

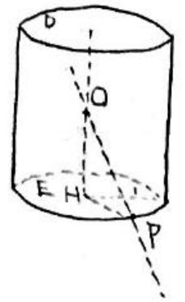
$H(k, k, -2k)$ とすると Hは平面ABC上より

$k+k+4k+1=0$

$6k=-1 \quad \therefore k=-\frac{1}{6}$

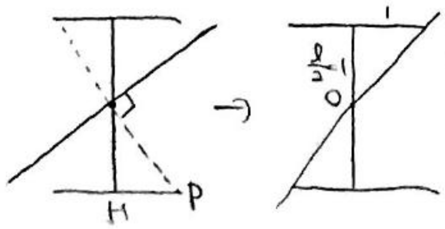
$\therefore H\left(-\frac{1}{6}, -\frac{1}{6}, \frac{1}{3}\right)$

$\boxed{3}$



$\tan(90°-\theta) = \frac{1}{\tan\theta} = \frac{1}{\frac{l}{2}}$

$\therefore \tan\theta = \frac{1}{2}l$

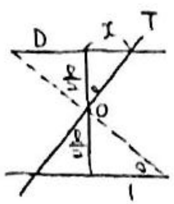


図より

$0 < \frac{l}{2} \leqq 1$

$\Leftrightarrow 0 < l \leqq 2$

面B.



図より

$1 : \frac{l}{2} = \frac{l}{2} : x$

$x = \frac{l^2}{4}$

よって

(交線とOとの距離)

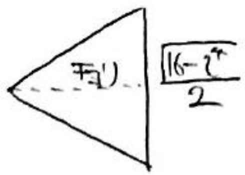
$= \sqrt{\frac{l^2}{4} + \frac{l^4}{16}}$

$= \frac{l}{4}\sqrt{4+l^2}$

(交線の長さ)

$= 2\sqrt{1 - \frac{l^4}{16}} = \frac{\sqrt{16-l^4}}{2}$

断面.



$S = \frac{l}{4}\sqrt{4+l^2} \cdot \frac{\sqrt{16-l^4}}{4} \times \frac{1}{2}$

$= \frac{l}{16}(4+l^2)\sqrt{4-l^2}$

$= \frac{1}{16}\sqrt{-l^8-4l^6+16l^4+16l^2}$

$l^2 = t$ として

$g(t) = -t^4-4t^3+16t^2+64t$

$g'(t) = -4t^3-12t^2+32t+64$

$= -4(t+4)(t^2-t-4)$

増減

t	0	$\cdots$	$\frac{1+\sqrt{17}}{2}$	$\cdots$	4
$g'(t)$	$\times$	$+$	0	$-$	
$g(t)$	$\times$	$\nearrow$		$\searrow$	0

$l^2 = \frac{1+\sqrt{17}}{2}$ で最大

$\boxed{4}$

問1

$_{12}C_4 \times {}_8C_4 \times {}_4C_4 \div 3!$

$= \frac{12\cdot11\cdot10\cdot9}{4\cdot3\cdot2\cdot1} \cdot \frac{8\cdot7\cdot6\cdot5}{4\cdot3\cdot2\cdot1} \cdot \frac{1}{6}$

$= 165 \cdot 35$

$= 5775$

問2.

$({}_6C_2 \times {}_4C_2 \times {}_2C_2 \div 3!)^2 \times 3!$

$= 15 \cdot 15 \cdot 6$

$= 1350$

問3.

BBBB BBGG GGGG … ①

BBBB BGGG BGGG … ②

BBBG BBBG GGGG … ③

①

$_6C_4 \cdot {}_6C_4 = 225$

②

$_6C_4 \cdot {}_2C_1 \cdot {}_6C_3 \div 2! = 300$

③

②と同じ 300

よって

$5775 - 225 - 300 - 300$

$= 4950$

$$\begin{array}{r} 5776 \\ 825 \\ \hline 4951 \end{array}$$