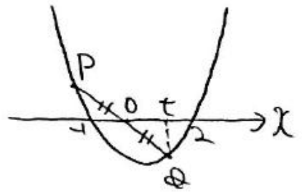
2012 日本医科大

$\boxed{1}$

(1)

(a)



$Q(t, t^2-t-2)$

$P(-t, t^2+t-2)$

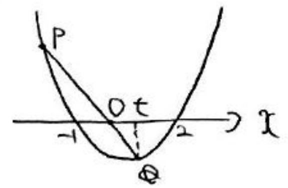
$\downarrow$

$t^2-t-2=-(t^2+t-2)$

$2t^2=4 \quad \therefore t=\sqrt{2}$

$Q(\sqrt{2}, -\sqrt{2}) \quad \therefore \underline{y=-x}$ //

(b)



$Q(t, t^2-t-2)$.

$P(-2t, 4t^2+2t-2)$

$\downarrow$

$4t^2+2t-2=-2(t^2-t-2)$

$6t^2=6 \quad \therefore t=1$

$Q(1,-2) \quad \therefore \underline{y=-2x}$ //

(c)

$S=\frac{1}{6}(1-(-2))^3$

$=\underline{\frac{9}{2}}$ //

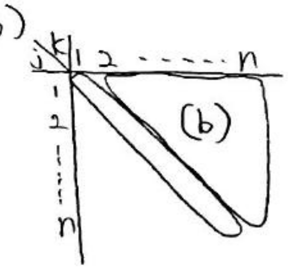
(2)

(a)

$(1+2+\cdots+n)(1+2+\cdots+n)$

$=\underline{\frac{1}{4}n^2(n+1)^2}$ //

(b)



$(b)=\frac{1}{2}\{(a)-(1^2+2^2+\cdots+n^2)\}$

$=\frac{1}{2}\{\frac{1}{4}n^2(n+1)^2-\frac{1}{6}n(n+1)(2n+1)\}$

$=\frac{1}{24}n(n+1)\{3n(n+1)-2(2n+1)\}$

$=\frac{1}{24}n(n+1)\underbrace{(3n^2-n-2)}_{(3n+2)(n-1)}$

$=\frac{1}{24}(n-1)n(n+1)(3n+2)$

$(c)=(b)-(1\cdot2+2\cdot3+\cdots+(n-1)n)$

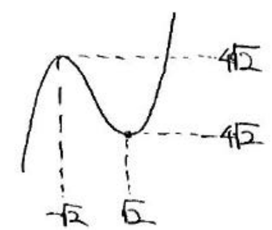
$=(b)-\underbrace{\sum_{k=1}^{n-1}k(k+1)}_{\frac{1}{3}(n-1)n(n+1)}$

$=\frac{1}{24}(n-1)n(n+1)(3n+2-8)$

$=\underline{\frac{1}{8}(n-2)(n-1)n(n+1)}$. //

(3)

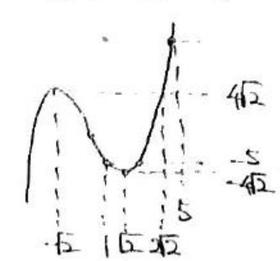
(a) $f'(x)=3(x^2-2)$



$f(\sqrt{2})=2\sqrt{2}-6\sqrt{2}=\underline{-4\sqrt{2}}$ //

(b) $-4\sqrt{2}\leqq a\leqq 4\sqrt{2}$

(c)



$\underline{-4\sqrt{2}<a\leqq-5}$ //

$\boxed{2}$

(1) $\overrightarrow{A_kA_{k+1}}$

$=\begin{pmatrix}\cos(\frac{\alpha}{2}+k\alpha)-\cos(-\frac{\alpha}{2}+k\alpha)\\ \underbrace{\sin(\frac{\alpha}{2}+k\alpha)}_{\sin k\alpha\cos\frac{\alpha}{2}+\cos k\alpha\sin\frac{\alpha}{2}}-\underbrace{\sin(-\frac{\alpha}{2}+k\alpha)}_{\sin k\alpha\cos\frac{\alpha}{2}-\cos k\alpha\sin\frac{\alpha}{2}}\end{pmatrix}$

$=\begin{pmatrix}-2\sin\frac{\alpha}{2}\sin k\alpha\\ 2\sin\frac{\alpha}{2}\cos k\alpha\end{pmatrix}$

$=2\sin\frac{\alpha}{2}\begin{pmatrix}\cos(k\alpha+\frac{\pi}{2})\\ \sin(k\alpha+\frac{\pi}{2})\end{pmatrix}$

$\therefore \underline{r=2\sin\frac{\alpha}{2}}$ //, $\theta_k=k\alpha+\frac{\pi}{2}$ より

$\underline{\theta_0=\frac{\pi}{2},\ \theta_1=\alpha+\frac{\pi}{2},\ \theta_2=2\alpha+\frac{\pi}{2}}$ //

(2) 与式のx座標に関して

$\cos\frac{\alpha}{2}-2\sin\frac{\alpha}{2}\sum_{k=0}^{n}\sin k\alpha=\cos(n\alpha+\frac{\alpha}{2})$

$\Leftrightarrow \cos\frac{\alpha}{2}-\cos(n+\frac{1}{2})\alpha$

$=2\sin\frac{\alpha}{2}\sum_{k=0}^{n}\sin k\alpha$

$\therefore \underline{\sum_{k=0}^{n}\sin k\alpha=\frac{\cos\frac{\alpha}{2}-\cos(n+\frac{1}{2})\alpha}{2\sin\frac{\alpha}{2}}}$ //

y座標に関して

$-\sin\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\sum_{k=0}^{n}\cos k\alpha$

$=\sin(n+\frac{1}{2})\alpha$

$\therefore \underline{\sum_{k=0}^{n}\cos k\alpha=\frac{\sin\frac{\alpha}{2}+\sin(n+\frac{1}{2})\alpha}{2\sin\frac{\alpha}{2}}}$ //

(3)

$$\sum_{k=0}^{n}\cos k\alpha \sin k\alpha$$

$$=\frac{1}{2}\sum_{k=0}^{n}\sin k\cdot 2\alpha$$

$$=\frac{\cos\alpha-\cos(2n+1)\alpha}{4\sin\alpha}$$

(4)

与

$$=\lim\frac{1}{n}\sum(\alpha^2\cos^2 k\alpha + \alpha\sin 2k\alpha+\sin^2 k\alpha)$$

$$=\lim\frac{1}{n}\sum\left\{\frac{\alpha^2}{2}(1+\cos 2k\alpha)+\alpha\sin 2k\alpha+\frac{1}{2}(1-\cos 2k\alpha)\right\}$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum\left\{\frac{\alpha^2+1}{2}+\alpha\sin 2k\alpha+\frac{\alpha^2-1}{2}\cos 2k\alpha\right\}\cdots ①$$

ここで

$$-1\leqq -\cos(2n+1)\alpha\leqq 1$$

$$\Leftrightarrow \frac{\cos\alpha-1}{2\sin\alpha}\leqq\sum_{k=0}^{n}\sin 2k\alpha\leqq\frac{\cos\alpha+1}{2\sin\alpha}$$

より

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n}\sin 2k\alpha=0$$

同様に

$$\sum_{k=0}^{n}\cos 2k\alpha=\frac{\sin\alpha+\sin(2n+1)\alpha}{2\sin\alpha}$$

から

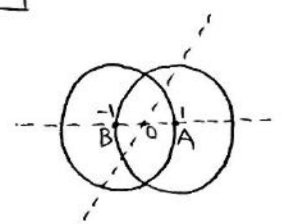
$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n}\cos 2k\alpha=0$$

なので①は

$$\lim\left\{\frac{1}{n}\cdot\frac{\alpha^2+1}{2}\cdot n+0+0\right\}$$

$$=\frac{\alpha^2+1}{2}$$

$\boxed{3}$



A(1.0.0), B(−1.0.0)とすると

$$\overrightarrow{OP}=\begin{pmatrix}1\\0\\0\end{pmatrix}+\overrightarrow{AP}$$

$$\overrightarrow{OQ}=\begin{pmatrix}-1\\0\\0\end{pmatrix}+\overrightarrow{BQ}$$

$$\therefore \overrightarrow{OX}=\overrightarrow{OP}+\overrightarrow{PQ}=\overrightarrow{AP}+\overrightarrow{BQ}$$

$\overrightarrow{AP}$, $\overrightarrow{BQ}$はそれぞれ長さ2の(以下)すべてのベクトルをとる。

よって$\overrightarrow{OX}$は長さ4以下のすべてのベクトルとなるのでXは半径4の球面および内部。体積は

$$\frac{4\pi}{3}4^3=\frac{256\pi}{3}$$

(2)

$$\overrightarrow{OY}=\frac{1}{3}\overrightarrow{OX}+\frac{1}{3}\overrightarrow{OR}$$

$\frac{1}{3}\overrightarrow{OX}$が長さ$\frac{4}{3}$以下のすべてのベクトル。Yが$K_1$に属するには

$$\overrightarrow{OR}=\begin{pmatrix}3\\0\\0\end{pmatrix}+\vec{\alpha}$$

$\vec{\alpha}$は長さ2以下のすべてのベクトルになるようにすればよい。

Rは半径2の球面および内部より体積は

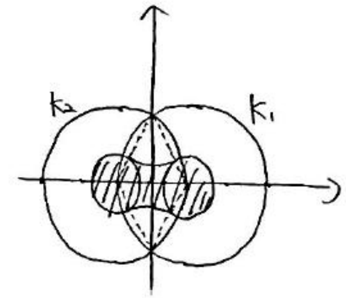
$$\frac{4\pi}{3}2^3=\frac{32\pi}{3}$$

(3)

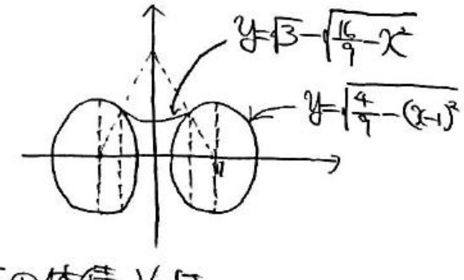
$$\overrightarrow{OY}=\frac{1}{3}\overrightarrow{OX}+\overrightarrow{OS}$$

とおくとSは、K_1に属する点から距離$\frac{4}{3}$以内の集合となる。

Sの体積を3^3倍すれば答え。



この斜線部分のx軸まわりの回転体



Sの体積 Vsは

$$V_S=\frac{4\pi}{3}\left(\frac{2}{3}\right)^3+2\left\{\int_{\frac{2}{3}}^{1}\pi\left\{\frac{4}{9}-(x-1)^2\right\}dx+\int_{0}^{\frac{2}{3}}\pi\left\{\sqrt{3}-\sqrt{\frac{16}{9}-x^2}\right\}^2dx\right\}$$

$$=\frac{32\pi}{81}+2\pi\left[\frac{4}{9}x-\frac{1}{3}(x-1)^3\right]_{\frac{2}{3}}^{1}+2\pi\int_0^{\frac{2}{3}}\left\{\frac{43}{9}-x^2-2\sqrt{3}\sqrt{\frac{16}{9}-x^2}\right\}dx$$

$$=\cdots$$

$$=\frac{338}{81}\pi-\frac{16\sqrt{3}}{27}\pi^2$$

$$\therefore V_R=\frac{338}{3}\pi-16\sqrt{3}\pi^2$$